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Feasible Power-Flow Solution Analysis of DC Microgrid Considering Distributed Generations under MPPT Control

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***Abstract*—For a DC microgrid with constant power loads (CPLs), the existence of a feasible power-flow solution, which is usually difficult to analyze, is a necessary condition for the correct operation of systems. In this paper, the solvability of power-flow equation of DC microgrid with CPLs is analyzed, where a majority of distributed generations (DGs) are under MPPT control while other DGs are under droop control. At first, this paper builds a power-flow mathematical model of the DC microgrid. Secondly, three solvability conditions are proposed with the first one being applied to ensure the power-flow equation has a solution, while the other two applying to ensure the solution within the given voltage deviation. The first analytical condition (Theorem 2) is obtained by applying Brouwer fixed-point theorem. Compared with the existing results, this obtained sufficient condition is less conservative. Furthermore, the other two sufficient solvability conditions (Theorem 3 and 4) can guarantee the equilibrium not only exists, but also the voltage deviation is within an acceptable range. Finally, case studies verify the correctness of the proposed theorems. The obtained conditions provide a guidance for establishing a dependable DC microgrid.**

***Index Terms*—DC microgrid, solvablity, constant power load, MPPT, power-flow equation, Brouwer fixed-point theorem.**

I. INTRODUCTION

Microgrids, which contain renewable generators, have been regarded as essential and complemental role in the power system. Microgrids are usually divided into two types: DC and AC [1][2]. DC microgrids provide high transmission efficiency, high reliability and flexibility [2]-[6] over their AC counterparts. Hence, in recent decades, DC microgrids are widely utilized in the application areas of aviation and traffic to eliminate redundant power conversion losses, for instance, space crafts, aircrafts and electric vehicles [7]-[10]. In DC microgrids, to improve energy efficiency and achieve maximum power output, the renewable energy DGs are usually under MPPT control algorithm (MPPT-DGs). Meanwhile, the traditional DGs are normally under droop control (Droop-DGs), supporting the voltage of DC system.

Loads in DC microgrids are mainly connected to the DC bus through DC-DC or DC-AC converters, letting the loads to behave as constant power loads (CPLs) [7]-[9][11]. It is well known that CPLs can easily cause the DC microgrids unstable [10][12]. Most stability studies are based on small signal stability analysis [8][9][13]-[22]. It is necessary to know the equilibrium of the DC microgrid to establish an equivalent linearized model around the equilibrium for stability analysis [10][12]. In fact, the existence of an equilibrium is a prerequisite for the stability of the DC microgrid. However, due to the increasing of the CPLs, DC microgrids might lose the equilibrium because of the transmission loss, resulting in the voltage collapse [12][21][27]-[28]. Then, it is critical to find methodologies that ensure the existence of the equilibrium.

DC microgrids can be divided into two types by the complexity of power-flow equation, namely star-connection DC microgrids (system with n DGs and one CPL) and meshed DC microgrids (system with n DGs and m CPLs) [27]. The power-flow equation of star-connection DC microgrids is a quadratic equation with one unknown which solution can be obtained by quadratic formula [7][9][29]. The power-flow equation of meshed DC microgrids is complicated because it is a multi-dimensional quadratic equation (MDQE) with multiple unknowns [7][11][12][21].

Due to the existence of CPLs and MPPT-DGs, the power-flow equation of DC microgrid is a nonlinear equation, which makes the process of solving this equation difficult [23]. Therefore, in order to determine the feasible solution, many methods have been used [24]-[26]. In [24], it formulates the power-flow analyses for both AC and DC microgrid considering droop control and virtual impedance, which improved the calculation accuracy. [25] proposes a linear approximation method based on a Taylor series expansion to solve power-flow equation of DC power grids, leading to an explicit solution and avoiding the use of an iterative process. Newton-Raphson method is used in [26] to solve power-flow bidirectionally between the AC and DC subgrids. However, [24] and [26] only consider Droop-DGs, the influence of MPPT-DGs are not considered. In addition, the solvability of the power-flow equation is the prerequisite for the use of these methods.

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Hence, this paper analyzes the solvability problem of the power-flow equation.

There are three general approaches to the solvability of the MDQE, that is, "quadratic form method" [12], "fixed-point theorem method" [27]-[33] and "nested interval theorem method" [11]. The sum-equation of all power-flow equations is a quadratic form and its solvability can be determined by completing the square. If the system has a equilibrium, then the sum-equation is solvable. Based on this idea, a necessary condition is obtained in [12]. To obtain sufficient solvability conditions, several methods based on "fixed-point theorem method" are proposed. These methods firstly aim at transforming the MDQE into the equation taking form as $f(x) = x$, where f is a constructive mapping [27][34]. Then if $f(x)$ is a "continuous self-mapping", "concave self-mapping" and "contraction self-mapping" respectively, according to three fundamental theorems concerning fixed points of Brouwer [31][33], Tarski [27] and Banach [28]-[29][32][35], there exists a x^* satisfies $f(x^*) = x^*$. Based on this method, several conditions are proposed, and the sufficient conditions obtained in [27] and in [28] can guarantee the uniqueness of the solution while the condition obtained in [31] and [33] can not. The third method "nested interval theorem method" has been applied to analyze the power-flow solution of DC microgrids under droop control in [11], they transform the solvability problem of the MDQE into a convergence problem of two monotone infinite sequences to get the sufficient condition. The existing conditions mainly aim at DC microgrids with only Droop-DGs. Only the condition obtained in [28] can be applicable to DC microgrids with a few MPPT-DGs while the condition is still conservative. Therefore, it is important to extend the analysis regarding the existence of the feasible power-flow solution of DC microgrids with a large amount of MPPT-DGs.

This paper aims to analyze the existence of the feasible power-flow solution of the DC microgrid contains MPPT-DGs, and propose a stronger existence condition. The main contributions of this paper are summarized as follows.

1) In order to ensure the DC microgrid contain CPLs and MPPT-DGs has a feasible power-flow solution, an analytical solvability condition is derived based on Brouwer fixed-point theorem. This condition is less conservative than existing results.

2) Two solvability conditions are obtained not only to guarantee the existence of the equilibrium, but also ensure the feasible solution is within a given voltage deviation ε.

3) The derived feasible conditions that guarantee the power-flow equation's solvability can be illustrated in a two-dimensional plane. If the system parameters are in the feasible set, the system exists the equilibrium, providing a guidance for establishing a dependable DC microgrid.

The rest of the paper is organized as follows. Section II introduces preliminaries and notations. Section III presents the power-flow equation of the DC microgrid considering distributed Generations under MPPT Control. The main results aimed to ensure the existence of the feasible power-flow solution of the DC microgrid are obtained in section IV. Section V presents the simulation results and proves the correctness of the main results. Finally, in section VI, we draw the conclusion.

II. Preliminaries and Notations

Notations. $\mathbb{R}, \mathbb{R}_+, \mathbb{R}^m, \mathbb{R}^{m\times n}$ represent the set of the real numbers, positive real numbers, real m-dimensional vector and real $m \times n$ matrix, respectively. O represents the zero matrix with proper dimension. Let $M \in \mathbb{R}^{m\times m}$ be a symmetric matrix, M is positive definite (denoted by $M > 0$) if and only if $x^{\mathrm{T}}Mx > 0, \forall x \in \mathbb{R}^m$ and $x^{\mathrm{T}}x \neq 0$, where x^{T} denotes the transpose of x. Let $A = [a_{ij}], B = [b_{ij}] \in \mathbb{R}^{m\times n}$, $A \succ B$ denotes $a_{ij} > b_{ij}$. A is called positive if $A \succ O$. Define $\|A\|_\infty = \max\limits_{1\le i\le n} \left\{\sum_{j=1}^{n} |a_{ij}|\right\}$. Let $x = [x_1\ x_2\ ...\ x_m]^{\mathrm{T}}$ and $x_i \neq 0$, we define $x^{-1} = \left[x_1^{-1}\ x_2^{-1}\ ...\ x_m^{-1}\right]^{\mathrm{T}}$ and $[\![x]\!] = diag\{x_i\}$. Meanwhile, $1_m(0_m)$ is defined as the m-dimensional vector that all elements are 1(0). Let $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$, $A \in \mathbb{R}^{n\times n}$, $D \in \mathbb{R}^{m\times m}$, if A is invertible, then $S_A = D - CA^{-1}B$ represents the Schur complement of M .

Definition 1. Let $A \in \mathbb{R}^{m\times m}$, when its off–diagonal elements are zero or negative, we call it a Z-matrix. Meanwhile, it is also an M-matrix if and only if the the eigenvalues of A are in the right half-plane [36] [37].

Lemma 1. If matrix A is an irreducible M-matrix, then $A^{-1} \succ O$ [38].

Lemma 2. Let $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ be an invertible matrix. If $A \in \mathbb{R}^{m\times m}, D \in \mathbb{R}^{n\times n}$ are all invertible, then the following equation can be obtained [39]

$$M^{-1} = \begin{bmatrix} F & -A^{-1}BE \\ -D^{-1}CF & E \end{bmatrix}$$

where $E = (D - CA^{-1}B)^{-1}$, $F = (A - BD^{-1}C)^{-1}$.

Lemma 3. (Brouwer fixed-point theorem [40].) Given that set $D \subset \mathbb{R}^n$ is compact and convex, and that $f(x) : D \to D$ is a continuous function, then there exists some $x \in D$ such that $f(x) = x$; that is, x is a fixed point.

III. Power-Flow Equation of DC-Microgrids Considering MPPT-DGs

In this article, we consider a DC microgrid power distribution system with h Droop-DGs, m MPPT-DGs, n CPLs which is shown in Fig.1. This system is composed by three main parts, that is, power sources, loads and cables. In the case of low voltage system, the cable is purely resistive, and the loads are assumed to be CPLs. According to graph theory, the equivalent structure of the system is equivalent to the graph shown in Fig.1.b, where loads and sources are considered as nodes, and cables are considered as edges. Furthermore, we assume that the equivalent graph of the system and the subgraph induced by the load nodes are strongly connected.

Applying the Ohm's and Kirchoff's laws, the current injected into the transmission network by each node can be described as follows

$$\begin{bmatrix} i_S \\ i_M \\ i_L \end{bmatrix} = \begin{bmatrix} Y_{SS} & Y_{SM} & Y_{SL} \\ Y_{MS} & Y_{MM} & Y_{ML} \\ Y_{LS} & Y_{LM} & Y_{LL} \end{bmatrix} \begin{bmatrix} u_S \\ u_M \\ u_L \end{bmatrix} = Y \begin{bmatrix} u_S \\ u_M \\ u_L \end{bmatrix} \quad (1)$$

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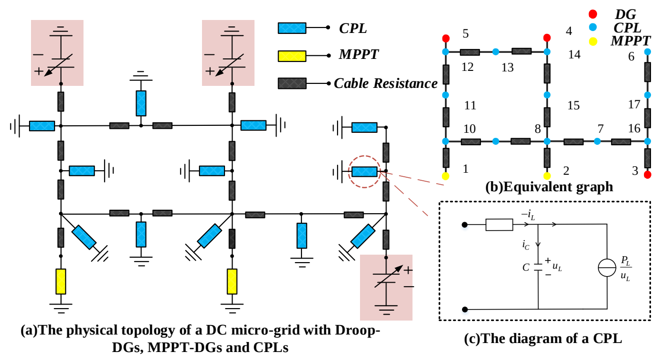

Fig. 1. A DC microgrid power distribution system with *h* converters (DGs), *m* MPPTs, *n* CPLs. In (a), the black blocks represent the cable resistance and the DGs, MPPTs and CPLs are the red, yellow and blue blocks, respectively. In (b), it is an equivalent graph of (a), the red, blue and yellow points figure the DGs, CPLs and MPPTs, respectively. In (c), the control diagram is shown.

where $Y \in \mathbb{R}^{(h+m+n)\times(h+m+n)}$ is the symmetric admittance matrix of the system, and $i_S \in \mathbb{R}^h$, $i_M \in \mathbb{R}^m$ and $i_L \in \mathbb{R}^n$ are the current vectors of Droop-DGs, MPPT-DGs and CPLs injecting to the network, respectively. $u_S \in \mathbb{R}^h, u_M \in \mathbb{R}^m$ and $u_L \in \mathbb{R}^n$ represent the voltage vectors of Droop-DGs, MPPT-DGs and CPLs respectively.

For MPPT-DGs and CPLs, the power-balance equations are given by

$$\begin{bmatrix}[\![u_M]\!] & O \\ O & [\![u_L]\!]\end{bmatrix}\begin{bmatrix}i_M \\ i_L\end{bmatrix} = \begin{bmatrix}P_M \\ -P_L\end{bmatrix} \tag{2}$$

where $P_M \in \mathbb{R}^m$ represents the output power vector of MPPT-DGs and $P_L \in \mathbb{R}^m$ is the consumed power vector of CPLs. The powers injected into the network by MPPT-DGs and CPLs are positive and negative respectively, since CPL's actual current direction and reference are opposite.

When the system is in a steady state, the output voltage of the sources can be described as

$$u_S = u_{ref}1_h - Ki_S \tag{3}$$

where $K = diag\{k_i\}$, $u_{ref} \in \mathbb{R}_+$ is the voltage reference and k_i represents the droop gain of the i-th DG. Since K is a positive definite diagonal matrix, therefore K is invertible.

Substituting (3) into (1), we have

$$u_S = S_k K^{-1} u_{ref} 1_h - S_k \begin{bmatrix}Y_{SM} & Y_{SL}\end{bmatrix}\begin{bmatrix}u_M \\ u_L\end{bmatrix} \tag{4}$$

where $S_k = (Y_{SS} + K^{-1})^{-1}$. Then, substituting (4) into (1), it yields

$$\begin{aligned}\begin{bmatrix}i_M \\ i_L\end{bmatrix} &= \begin{bmatrix}Y_{MS} \\ Y_{LS}\end{bmatrix} S_k K^{-1} u_{ref} 1_h \\ &- \begin{bmatrix}Y_{MS} \\ Y_{LS}\end{bmatrix} S_k \begin{bmatrix}Y_{SM} & Y_{SL}\end{bmatrix}\begin{bmatrix}u_M \\ u_L\end{bmatrix} \\ &+ \begin{bmatrix}Y_{MM} & Y_{ML} \\ Y_{LM} & Y_{LL}\end{bmatrix}\begin{bmatrix}u_M \\ u_L\end{bmatrix} \\ &= \begin{bmatrix}Y_{MS} \\ Y_{LS}\end{bmatrix} S_k K^{-1} u_{ref} 1_h + Y_{eq}\begin{bmatrix}u_M \\ u_L\end{bmatrix}\end{aligned} \tag{5}$$

Define Y_{eq} as

$$Y_{eq} = \begin{bmatrix}Y_{MM} & Y_{ML} \\ Y_{LM} & Y_{LL}\end{bmatrix} - \begin{bmatrix}Y_{MS} \\ Y_{LS}\end{bmatrix} S_k \begin{bmatrix}Y_{SM} & Y_{SL}\end{bmatrix} \tag{6}$$

It is noted that Y_{eq} is the Schur complement of block $Y_{SS} + K^{-1}$ of matrix T where

$$T = \begin{bmatrix}Y_{SS} + K^{-1} & Y_{SM} & Y_{SL} \\ Y_{MS} & Y_{MM} & Y_{ML} \\ Y_{LS} & Y_{LM} & Y_{LL}\end{bmatrix} \tag{7}$$

Substituting (5) into (2), the power-flow equation is obtained as follows

$$\begin{aligned}&\begin{bmatrix}[\![u_M]\!] & \\ & [\![u_L]\!]\end{bmatrix}\left(\begin{bmatrix}Y_{MS} \\ Y_{LS}\end{bmatrix} S_k K^{-1} u_{ref} 1_h + Y_{eq}\begin{bmatrix}u_M \\ u_L\end{bmatrix}\right) \\ &= \begin{bmatrix}P_M \\ -P_L\end{bmatrix}\end{aligned} \tag{8}$$

Clearly, the power-flow equation (8) is a MDQE.

Multiplied by $Y_{eq}^{-1}\begin{bmatrix}[\![u_M]\!] & \\ & [\![u_L]\!]\end{bmatrix}^{-1}$, (8) becomes

$$\begin{aligned}\begin{bmatrix}u_M \\ u_L\end{bmatrix} &= -Y_{eq}^{-1}\begin{bmatrix}Y_{MS} \\ Y_{LS}\end{bmatrix} S_k K^{-1} u_{ref} 1_h \\ &- Y_{eq}^{-1}\begin{bmatrix}-[\![P_M]\!] & \\ & [\![P_L]\!]\end{bmatrix}\begin{bmatrix}u_M \\ u_L\end{bmatrix}^{-1}\end{aligned} \tag{9}$$

Definition 2. A nonlinear equation is solvable if this equation has a positive real solution.

Remark 1. (9) is the power-flow equation of the system. It is noted that, if and only if, the nonlinear equation (9) is solvable with given values of K, u_{ref}, P and Y, then the whole system has an equilibrium. To derive the solvability condition for (9), we firstly investigate the properties for Y_{eq}.

Theorem 1: For a strongly connected DC microgrid, the following two statements hold.

(1) Y_{eq}^{-1} is a positive matrix, that is $Y_{eq}^{-1} \succ O$.

(2) $-Y_{eq}^{-1}\begin{bmatrix}Y_{MS} \\ Y_{LS}\end{bmatrix} S_k K^{-1} u_{ref} 1_h = u_{ref} 1_{m+n}$.

Proof. Firstly, for the statement (1) in Theorem 1, since the DC microgrid is strongly connected, Y is an irreducible positive semi-definite Z-matrix with only one zero eigenvalue [27].

According to (7), T is an irreducible symmetric Z-matrix, as well as Y. For any non-zero $x \in \mathbb{R}^{h+m+n}$, we obtain

$$x^{\mathrm{T}}Tx = \begin{cases} > 0, \forall x \in span\{1_{h+m+n}\} \\ \geq x^{\mathrm{T}}Yx > 0, \forall x \in \mathbb{R}^{h+m+n} - span\{1_{h+m+n}\}\end{cases}$$

Hence, T is a positive definite matrix. Since T is a positive definite Z-matrix, according to Definition 1, T is also an irreducible M-matrix. Then, according to Lemma 1, T^{-1} is a positive matrix.

By invoking Lemma 2, since Y_{eq} is a Schur complement of T, T^{-1} is given by

$$T^{-1} = \begin{bmatrix}* & * \\ * & Y_{eq}^{-1}\end{bmatrix} \tag{10}$$

Since T^{-1} is a positive matrix, that is, all elements of this matrix are larger than zero, then Y_{eq}^{-1} is positive and irreducible. Thus the statement (1) is proved.

Since Y is a Laplacian matrix, $Y1_{n+m} = 0_{n+m}$, i.e.,

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$$\begin{cases} Y_{SS}1_h + Y_{SM}1_m + Y_{SL}1_n = 0_h \\ Y_{MS}1_h + Y_{MM}1_m + Y_{ML}1_n = 0_m \\ Y_{LS}1_h + Y_{LM}1_m + Y_{LL}1_n = 0_n \end{cases} \tag{11}$$

To prove the statement (2), we firstly calculate its variant as follows

$$\begin{aligned} & u_{ref}1_{m+n} + Y_{eq}^{-1}\begin{bmatrix} Y_{MS} \\ Y_{LS} \end{bmatrix} S_k K^{-1} u_{ref} 1_h \\ & = u_{ref} Y_{eq}^{-1}\left(Y_{eq}1_{m+n} + \begin{bmatrix} Y_{MS} \\ Y_{LS} \end{bmatrix} S_k K^{-1} 1_h\right) \\ & = u_{ref} Y_{eq}^{-1}\left(Y_{eq}1_{m+n} + \left(\begin{bmatrix} Y_{MS} \\ Y_{LS} \end{bmatrix} - \begin{bmatrix} Y_{MS} \\ Y_{LS} \end{bmatrix} S_k Y_{SS}\right) 1_h\right) \end{aligned} \tag{12}$$

Notice that Y_{eq} is the Schur complement of T, the following is obtained

$$\begin{aligned} & Y_{eq}1_{m+n} + \left(\begin{bmatrix} Y_{MS} \\ Y_{LS} \end{bmatrix} - \begin{bmatrix} Y_{MS} \\ Y_{LS} \end{bmatrix} S_k Y_{SS}\right) 1_h \\ & = \left(\begin{bmatrix} Y_{MM} & Y_{ML} \\ Y_{LM} & Y_{LL} \end{bmatrix} - \begin{bmatrix} Y_{MS} \\ Y_{LS} \end{bmatrix} S_k \begin{bmatrix} Y_{SM} & Y_{SL} \end{bmatrix}\right) 1_{m+n} \\ & + \left(\begin{bmatrix} Y_{MS} \\ Y_{LS} \end{bmatrix} - \begin{bmatrix} Y_{MS} \\ Y_{LS} \end{bmatrix} S_k Y_{SS}\right) 1_h \\ & = \left(\begin{bmatrix} Y_{MM} & Y_{ML} \\ Y_{LM} & Y_{LL} \end{bmatrix} 1_{m+n} + \begin{bmatrix} Y_{MS} \\ Y_{LS} \end{bmatrix} 1_h\right) \\ & - \begin{bmatrix} Y_{MS} \\ Y_{LS} \end{bmatrix} S_k \left(\begin{bmatrix} Y_{SM} & Y_{SL} \end{bmatrix} 1_{m+n} + Y_{SS}1_h\right) \\ & = \begin{bmatrix} Y_{MM}1_m + Y_{ML}1_n + Y_{MS}1_h \\ Y_{LM}1_m + Y_{LL}1_n + Y_{LS}1_h \end{bmatrix} \\ & - \begin{bmatrix} Y_{MS} \\ Y_{LS} \end{bmatrix} S_k \left(Y_{SM}1_m + Y_{SL}1_n + Y_{SS}1_h\right) \end{aligned} \tag{13}$$

Substituting (11) to (13), we get

$$Y_{eq}1_{m+n} + \begin{bmatrix} Y_{MS} \\ Y_{LS} \end{bmatrix} S_k K^{-1} u_{ref} 1_h = 0_{m+n},$$

i.e.,

$$-Y_{eq}^{-1}\begin{bmatrix} Y_{MS} \\ Y_{LS} \end{bmatrix} S_k K^{-1} u_{ref} 1_h = u_{ref} 1_{m+n} \tag{14}$$

Thus, the statement (2) is proved, and the proof is completed.

According to Theorem 1, (8) becomes

$$f(u) = 1_{m+n} - Ju^{-1} = u \tag{15}$$

where $u = u_{ref}^{-2}\begin{bmatrix} u_M \\ u_L \end{bmatrix}, J = u_{ref}^{-2} A, A = Y_{eq}^{-1} P, P = \begin{bmatrix} -[\![P_M]\!] & \\ & [\![P_L]\!] \end{bmatrix}$.

The system has a feasible power-flow solution if and only if, for given A and u_{ref}, the nonlinear equation (15) admits a real solution. Next, we will derive the solvability conditions for (15).

IV. Existence Conditions of Power-Flow Solution for the DC Microgrid

A. The Existing Related Results

The existing results concerning the DC microgrid under droop control, whose power-flow equation is

$$[\![u_L]\!] B_{eq} u_L + [\![u_L]\!]\sigma + P_L = 0_n \tag{16}$$

where $B_{eq} = B_{LL} - B_{LG}\left(B_{GG} + K^{-1}\right)^{-1} B_{GL}$, $\sigma = -B_{LG}\left(I + KB_{GG}\right)^{-1} V$, and B_{LL}, B_{LG}, B_{GG}, B_{GL} are blocks of the admittance matrix B of system contains no MPPT-DGs. B_{eq} is a positive definite matrix, P_L is a positive vector and $\sigma \in \mathbb{R}^h$ (more details are in [11]).

To analyze the existence of the feasible power-flow solution of the DC microgrid, there are mainly three methods: "quadratic form method" [12], "fixed-point theorem method" [27]-[33], and "nested interval theorem method" [11].

The main procedure of the first method is to transfer the n-dimensional quadratic equation into a one-dimensional quadratic equation. If the sum-equation of all power-flow equations is not solvable, then (16) has no solution. Hence, a necessary solvability condition of (16) is that there is no diagonal matrix W making (17) holds [12].

$$\begin{cases} WB_{eq} + B_{eq}^{\mathrm{T}} W > 0 \\ 2 \times 1_n^T W P_L - \sigma^{\mathrm{T}} W (WB_{eq} + B_{eq}^{\mathrm{T}} W)^{-1} W\sigma > 0 \end{cases} \tag{17}$$

where $W = diag\{w_1, w_2, ..., w_n\}$ is the weighted sum matrix.

The first method can only obtain the necessary condition. Therefore, to obtain the sufficient solvability conditions, several methods based on fixed-point theorem are proposed.

The core procedure of fixed-point-theorem-based method is to change the MDQE's solvability problem into the problem of existence of the fixed-point of a mapping. Firstly, they transform (16) into (18).

$$x = f(x) = 1_n - Jx^{-1} \tag{18}$$

where $x = [\![u_{ref}1_n]\!]^{-1} u_L$, $J = u_{ref}^{-2} B_{eq}^{-1} [\![P_L]\!]$.

If there exists a convex closed set D such that $f(x) \in D$ and $\|\partial f/\partial x\| < 1$ for any $x \in D$, according to Banach's fixed-point theorem, there is a unique solution $x^* \in D$ such that $x^* = f(x^*)$. Then, based on this idea, a sufficient solvability condition is obtained as follows [28]

$$4\|J\|_\infty < 1 \tag{19}$$

However, the sufficient condition (19) based on Banach's fixed-point theorem needs to satisfy the strict condition of $\|\partial f/\partial x\| < 1$, hence the existence condition obtained in (19) is conservative.

Since $f(x)$ is a concave increasing function, based on Tarski fixed-point theorem, [27] gets the sufficient condition as follows

$$u_{ref} > \min\left\{2\sqrt{\left\|B_{eq}^{-1}[\![P_L]\!]\right\|_\infty}, \frac{\bar{\eta} + \underline{\eta}}{\sqrt{\bar{\eta}\underline{\eta}}}\right\} \tag{20}$$

where η and χ represent the Perron eigenvector and eigenvalue of $B_{eq}^{-1}[\![P_L]\!]$, $\underline{\eta} = \min\{\eta_i\}$, $\bar{\eta} = \max\{\eta_i\}$. It is noted that condition (20) is less conservative than (19).

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For the third method, since $f(x)$ is an increasing function, if there is a positive vector γ satisfies [11]

$$\gamma \prec f(\gamma) \tag{21}$$

then, the following is derived

$$\gamma \prec f(\gamma) \prec f(f(\gamma)) \prec f(1_n) \prec 1_n \tag{22}$$

Likewise, the following is obtained

$$a_1 \prec a_2 \prec \ldots \prec a_n \prec \cdots \prec b_n \prec \cdots \prec b_2 \prec b_1 \tag{23}$$

where sequences $a_{n+1} = f(a_n)$, $b_{n+1} = f(b_n)$, and $a_1 = x$, $b_1 = 1_m$. The sequences a_n and b_n consist of the nested intervals $[a_n, b_n]$. According to the nested interval theorem, there is a unique vector that satisfies $f(\gamma^*) = \gamma^*$, and $\gamma \prec \gamma^* \prec 1_n$, if (21) holds, i.e., the system has a feasible power-flow solution. It is worth mentioning that the condition (20) can also be derived from (21) (more details are in [11]).

B. Motivation

The solvability sufficient conditions obtained in [12] and [27] can guarantee the existence of the feasible power-flow solution of the DC microgrid under droop control. However, in the DC microgrid considering MPPT-DGs, since A in (15) is not a positive matrix, $f(x)$ will not be a monotonic function. Thus, the Tarski's fixed-point theorem and the nested interval theorem can not be applied in the DC microgrid with MPPT-DGs.

In this paper, we analyze the existence of power-flow solution of the DC microgrid considering MPPT-DGs. In particular, the following two questions will be addressed.

Q1. How to ensure that the DC system has a feasible power-flow solution?

Q2. For a given voltage deviation ε, how to ensure that the power-flow equation (15) has a feasible solution such that $u_{ref}(1-\varepsilon)1_{m+n} \leq u \leq u_{ref}(1+\varepsilon)1_{m+n}$?

C. Sufficient Condition for Existence of Equilibrium

The main results of this paper are as follows.

Define $A = \begin{bmatrix} -A_1 & A_2 \end{bmatrix}$, $A_1 \in \mathbb{R}^{(m+n)\times m}$, $A_2 \in \mathbb{R}^{(m+n)\times n}$. Define $J_1 = u_{ref}^{-2} A_1$, $J_2 = u_{ref}^{-2} A_2$. Then, $J = \begin{bmatrix} -J_1 & J_2 \end{bmatrix}$.

Theorem 2: If (24) holds, the equation (15) admits a solution in the convex closed set E as $E = \left\{ x \middle| (1-\sqrt{\frac{1-\|J\|_\infty}{3}})1_{m+n} \leq x \leq (1+\sqrt{\frac{1-\|J\|_\infty}{3}})1_{m+n} \right\}$.

$$1 - \|J\|_\infty - 3\left(\frac{\|J1_{m+n}\|_\infty}{2}\right)^{\frac{2}{3}} \geq 0 \tag{24}$$

Proof. If $f(x)$ is a continuous self-mapping in E, according to Lemma 3, there is a vector $x^* \in E$ such that $f(x^*) = x^*$. Therefore, we will prove that $f(x)$ is a self-mapping in E if (24) holds.

Define $t = \sqrt{\frac{1-\|J\|_\infty}{3}}$, then $3t^2 = 1 - \|J\|_\infty$ is obtained. And according to (24), it yields

$$2t^3 - \|J1_{m+n}\|_\infty \geq 0 \tag{25}$$

J_1 and J_2 are both positive matrices. The range of $f(x)$ yields for any $x \in E$

$$\begin{cases} f(x) < 1_{m+n} + \dfrac{J_1 1_m}{1-t} - \dfrac{J_2 1_n}{1+t} \\ f(x) > 1_{m+n} + \dfrac{J_1 1_m}{1+t} - \dfrac{J_2 1_n}{1-t} \end{cases} \tag{26}$$

Therefore, $f(x)$ is a self-mapping if the following inequalities hold.

$$\begin{cases} 1_{m+n} + \dfrac{J_1 1_m}{1-t} - \dfrac{J_2 1_n}{1+t} \leq (1+t)1_{m+n} \\ 1_{m+n} + \dfrac{J_1 1_m}{1+t} - \dfrac{J_2 1_n}{1-t} \geq (1-t)1_{m+n} \end{cases} \tag{27}$$

For the first inequality in (27), considering that

$$\begin{aligned} &1_{m+n} + \frac{J_1 1_m}{1-t} - \frac{J_2 1_n}{1+t} - (1+t)1_{m+n} \\ &= -\frac{1}{1-t^2}(t(1-t^2)1_{m+n} - (1+t)J_1 1_m \\ &+ (1-t)J_2 1_n) \\ &= -\frac{1}{1-t^2}(t(1-t^2)1_{m+n} + J_2 1_n - J_1 1_m \\ &- (J_1 1_m + J_2 1_n)t) \end{aligned} \tag{28}$$

Since J_1 and J_2 are all positive matrices, we have

$$\begin{cases} |J_1 1_m - J_2 1_n| = |J 1_{m+n}| < \|J 1_{m+n}\|_\infty \\ |J_1 1_m + J_2 1_n| < \|J_1 1_m + J_2 1_n\|_\infty = \|J\|_\infty \end{cases} \tag{29}$$

According to (25) and (29), the following is obtained

$$\begin{aligned} &t(1-t^2)1_{m+n} + J_2 1_n - J_1 1_m \\ &- (J_1 1_m + J_2 1_n)t \\ &\geq (t(1-t^2) - \|J1_{m+n}\|_\infty - \|J\|_\infty t)1_{m+n} \\ &= (t(1-t^2-\|J\|_\infty) - \|J1_{m+n}\|_\infty)1_{m+n} \\ &= (\frac{2}{3}t(1-\|J\|_\infty) - \|J1_{m+n}\|_\infty)1_{m+n} \\ &= (2t^3 - \|J1_{m+n}\|_\infty)1_{m+n} \\ &\geq 0_{m+n} \end{aligned} \tag{30}$$

The result in (30) shows that the first inequality of (27) holds. For the second inequality in (27), considering that

$$\begin{aligned} &1_{m+n} + \frac{J_1 1_m}{1+t} - \frac{J_2 1_n}{1-t} - (1-t)1_{m+n} \\ &= \frac{1}{1-t^2}(t(1-t^2))1_{m+n} \\ &+ (1-t)J_1 1_m - (1+t)J_2 1_n) \\ &= \frac{1}{1-t^2}(t(1-t^2))1_{m+n} + J_1 1_m \\ &- J_2 1_n - (J_1 1_m + J_2 1_n)t) \end{aligned} \tag{31}$$

According to the results in (25) and (29), it yields

$$\begin{aligned} &t(1-t^2))1_{m+n} + J_1 1_m - J_2 1_n - (J_1 1_m + J_2 1_n)t \\ &\geq (t(1-t^2) - \|J1_{m+n}\|_\infty - \|J\|_\infty t)1_{m+n} \\ &= (t(1-t^2-\|J\|_\infty) - \|J1_{m+n}\|_\infty)1_{m+n} \\ &= (2t^3 - \|J1_{m+n}\|_\infty)1_{m+n} \geq 0_{m+n} \end{aligned} \tag{32}$$

Then, (30) and (32) indicate that $f(x)$ is a self-mapping in E if (24) holds. The proof is completed.

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Remark 2: Theorem 2 provides an analytical solvability condition for the existence of feasible power-flow solution of the DC microgrid considering MPPT-DGs. Thus, (24) answers Q1.

D. Comparing with the Existing Result in [28]

In this section, the proposed analytical solvability condition (24) is compared with condition (7) in [28].

The condition (7) in [28] is a solvability condition obtained based on Banach's fixed point theorem, which is given in (19) in this paper.

Define $p = J_1 1_m$, $q = J_2 1_n$, and $p_i \in \mathbb{R}_+$, $q_i \in \mathbb{R}_+$ represent $i\text{-}th$ element of p and the $i\text{-}th$ element of q, respectively.

Since $J = \begin{bmatrix} -J_1 & J_2 \end{bmatrix}$, and according to (29), condition (19) and (24) are rewritten into (33)-(34).

$$4|J_1 1_m + J_2 1_n| < 1 \tag{33}$$

$$|J_1 1_m + J_2 1_n| + 3(\frac{|J_1 1_m - J_2 1_n|}{2})^{\frac{2}{3}} < 1 \tag{34}$$

Furthermore, (33)-(34) change into (35)-(36)

$$4(p_i + q_i) < 1 \tag{35}$$

$$(p_i + q_i) + 3(\frac{|p_i - q_i|}{2})^{\frac{2}{3}} < 1 \tag{36}$$

The comparison between condition (24) and (19) is illustrated in a two-dimensional plane(as shown in Fig.2). Clearly, condition (24) derived in this paper is less conservative than condition (19).

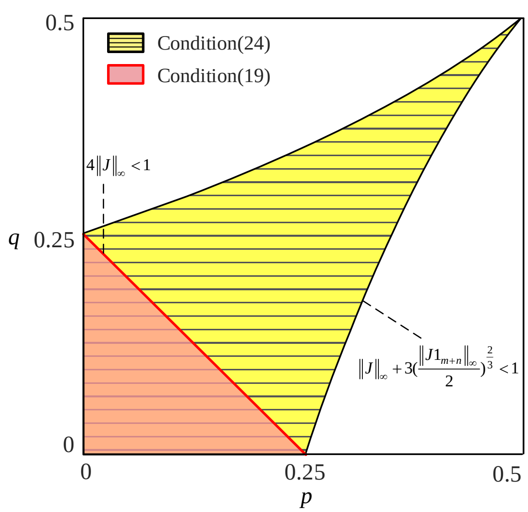

Fig. 2. The comparison of condition (24) and the condition in [28]

E. Solvability Conditions Considering Voltage Deviation

In section C, condition (24) is obtained, however, it may not guarantee that the equation (15) admits a solution for a given voltage deviation ε. Therefore, we will investigate Q2. The main results are as follows.

Define $D_\varepsilon = \{x | (1-\varepsilon)1_{m+n} \le x \le (1+\varepsilon)1_{m+n}\}$, and $\varepsilon \in (0,1)$. Then $E_\varepsilon \in \mathbb{R}^2$ is defined as the feasible region of the following inequalities.

$$\begin{cases} \dfrac{x}{1-\varepsilon} - \dfrac{y}{1+\varepsilon} \le \varepsilon \\ \dfrac{x}{1+\varepsilon} - \dfrac{y}{1-\varepsilon} \ge -\varepsilon \\ x, y > 0 \end{cases} \tag{37}$$

Theorem 3: For a given voltage deviation $\varepsilon \in (0,1)$, if (38) holds, (15) admits a solution in D_ε.

$$(p_i, q_i) \in E_\varepsilon, i \in \{1, \cdots, m+n\} \tag{38}$$

Proof. It is obvious that $f(x)$ is a self-mapping in D_ε if (38) holds. Therefore, according to Lemma 3, there exists $x \in D$ such that $f(x) = x$, i.e., (15) has a solution in D_ε. Thus, the proof is completed.

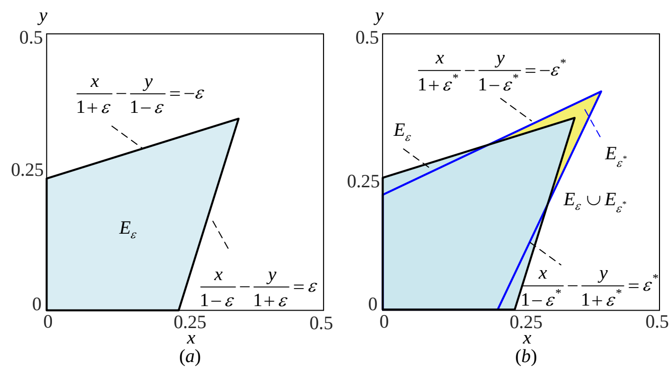

Fig. 3. The feasible region obtained by Theorem 4.

Remark 3: According to Theorem 3, for any $\varepsilon \in (0,1)$, there exists a E_ε (E_ε is shown as blue shadow in Fig.3.(a)) such that if $(p_i, q_i) \in E_\varepsilon$, (15) has a solution in D_ε. Let $\varepsilon^* < \varepsilon$, there must exist a E_{ε^*} such that (15) has a solution in D_{ε^*} when $(p_i, q_i) \in E_{\varepsilon^*}$. In other words, E_{ε^*} is another feasible condition for (15), therefore $E_\varepsilon \cup E_{\varepsilon^*}$ (shown in Fig 3.(b)) is the feasible condition of (15). Based on this idea, a less conservative condition is obtained as follows.

Define $F_\varepsilon = \lim\limits_{n \to \infty} . \bigcup\limits_{i=1}^{n} E_{\left(\frac{i\varepsilon}{n}\right)}$

Theorem 4: For a given voltage deviation $\varepsilon \in (0,1)$, if (39) holds, then (15) has a feasible solution in D_ε, i.e., the system has a feasible power-flow solution.

$$(p_i, q_i) \in F_\varepsilon \tag{39}$$

Remark 4: The approximate image of F_ε is presented in Fig.4. As shown in Fig.4, the black straight lines and red curves together form the envelope line of F_ε.

Remark 5: Theorem 3 and Theorem 4 are the solutions of Q2. In this paper, in order to solve Q1 and Q2, we describe the process of regulating voltage reference. As mentioned in Section C, Theorem 2 provides a feasible condition for the existence of power-flow solution of the DC microgrid considering MPPT-DGs, however, this obtained voltage reference u_{ref} can not meet the requirement of a given voltage deviation ε. Therefore, we propose Theorem 3 and Theorem 4 to ensure that the

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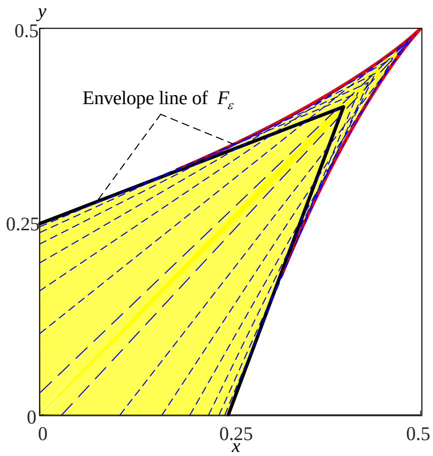

Fig. 4. The feasible region obtained by Theorem 4.

system's power-flow equation (15) has a feasible solution u such that $u_{ref}(1-\varepsilon)1_{m+n} \leq u \leq u_{ref}(1+\varepsilon)1_{m+n}$ with a given ε.

The flow chart, shown in Fig.5 below, summarizes the overall process of regulating voltage reference. The detailed process is discussed in part B of Section V.

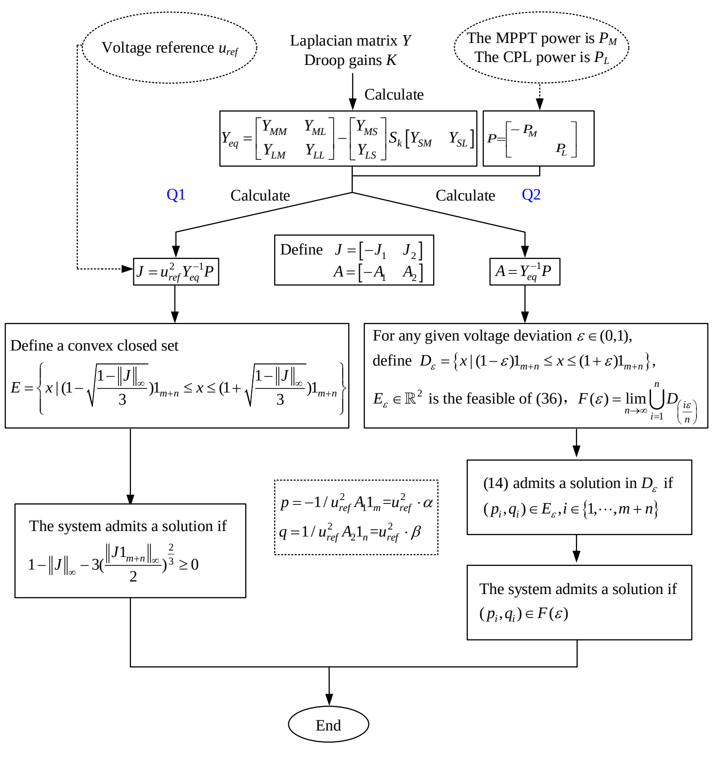

Fig. 5. The flow chart of the solution for Q1 and Q2.

Remark 6: In DC microgrids, the existence of a stable steady-state behavior, which is usually difficult to analyze, is crucial for the correct operation of DC distribution. This paper provides analytical existence conditions as a function of the system parameters (i.e., u_{ref}, Y, K, P_M and P_L), which provides a design guideline for planning a reliable DC microgrid.

F. Advantages of the Proposed Conditions Comparative Analysis with the Exiting Methods

In our work, we analyze the existence condition for the feasible power-flow solution of DC microgrid with MPPT-DGs.The advantages of the proposed approaches are highlighted as follows by making comparative analysis with existing methods.

1. The existence of a feasible power-flow solution is a prerequisite for the correct operation of power systems. For this problem, many existing literature such as [11][27], [28] and [31] only consider the Droop-DGs but ignore the influence of renewable generations (MPPT-DGs). In this paper, the existence conditions of the solution of the power-flow equation for the DC microgrid, which considers both Droop-DGs and MPPTDGs, are derived. Moreover, the feasible region of the proposed solvability condition is wider than some existing results in [28] and [35] ($4\left\|J\right\|_\infty < 1$).

2. For the power-flow analysis of DC microgrid, several methods such as Newton-Raphson method in [26] and approach without iterative method in [25] have been proposed to calculate the accurate solution of power-flow equation. However, only when the equilibrium of the system exits can these methods work, i.e., the DC power grid is operating under steady state conditions. In our work, we derive the solvability conditions (Theorem 2-4) to guarantee the existence of the equilibrium, which is a fundamental condition for power-flow analysis. Furthermore, the proposed analytical condition (24) is less conservative than (7) in [28]. This means that the proposed condition is stronger than existing result.

3. In addition, from a brand new perspective, we propose feasible conditions considering the voltage deviation (Theorem 3 and 4), which can be presented in a two-dimensional plane. This provides an intuitive guide for selecting system parameters that can guarantee the existence of equilibrium of DC microgrid.

V. Case Study

In this section, the correctness of theorems proposed in Section IV are verified. In order to verify the above analysis, we simulate a DC microgrid power distribution system with 20 Droop-DGs, 30 CPLs and 20 MPPT-DGs as shown in Fig.6. In Fig.6, red, blue and yellow points are Droop-DGs, CPLs and MPPT-DGs, respectively. Black lines represent the cables, and green numbers in this figure show resistance values (resistance values are given in Ω).

Simulations are performed using and MATLAB/Simulink. In simulations, the Droop-DGs are modeled as controlled voltage source based on (3). MPPT-DGs and CPLs are both modeled as controlled current sources based on equation (2).

Take the droop gain coefficients $k_1 = 1.5$, $k_2 = \cdots = k_{20} = 2$.

A. Comparison of Derived Condition and Condition in [24]

In this section, one case is designed to make sure that Theorem 2 is correct. Besides, a comparison is made to verify condition (24) is less conservative than (19).

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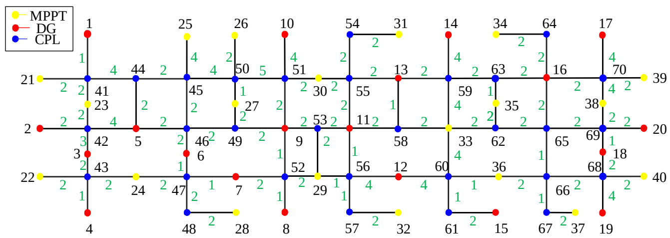

Fig. 6. A DC microgrid power distribution system with 20 converters (DGs), 20 MPPTs, 30 CPLs.

Let

$$\tau_1 = \|J\|_\infty + 3\left(\frac{\|J1_{m+n}\|_\infty}{2}\right)^{\frac{2}{3}} \quad (40)$$
$$\tau_2 = 4\,\|J\|_\infty\,.$$

According to condition (19) and (24), the system has a feasible solution if $\tau_1 < 1, \tau_2 < 1$. One case is designed to verify the correctness of Theorem 2.

Case 1: $P_{M,\max}$ is given as

$$P_{M,\max} = [100 \times 1_{10}^{\mathrm{T}}, 150 \times 1_5^{\mathrm{T}}, 200 \times 1_5^{\mathrm{T}},]^{\mathrm{T}} kW. \quad (41)$$

The power of MPPT-DGs are time-varying and partly random as shown in Fig.7.(a), and $P_M(t) \le P_{M,\max}$. The power of CPLs are as follows

$$\begin{cases} t < 0.2 \\ P_L = [100 \times 1_{10}^{\mathrm{T}}, 150 \times 1_{10}^{\mathrm{T}}, 200 \times 1_{10}^{\mathrm{T}},]^{\mathrm{T}} kW, \\ 0.2 \le t < 0.4 \\ P_L = [180 \times 1_{10}^{\mathrm{T}}, 230 \times 1_{10}^{\mathrm{T}}, 280 \times 1_{10}^{\mathrm{T}},]^{\mathrm{T}} kW. \end{cases} \quad (42)$$

In order to compare these two conditions, we take u_{refi} that can ensure $\tau_1 = \tau_2 = 0.999$. According to (40), we can quote the equations used to derive u_{refi} values as follows

$$u_{ref1} \ge \sqrt{\frac{\|A\|_\infty}{\tau_1} + 3(\kappa_1 + \kappa_2)} \quad (43)$$
$$u_{ref2} \ge 2\sqrt{\frac{\|A\|_\infty}{\tau_2}}.$$

where

$$\kappa_1 = 3\left(\sqrt[3]{\frac{\|A\|_\infty}{2\tau_1} + \kappa_3}\right)^2 \sqrt[3]{\frac{\|A\|_\infty}{2\tau_1} - \kappa_3},$$

$$\kappa_2 = 3\left(\sqrt[3]{\frac{\|A\|_\infty}{2\tau_1} - \kappa_3}\right)^2 \sqrt[3]{\frac{\|A\|_\infty}{2\tau_1} + \kappa_3},$$

$$\kappa_3 = \sqrt{\frac{\|A\|_\infty{}^2\tau_1 - \|A1_{m+n}\|_\infty{}^2}{4\tau_1^3}}.$$

The proof of (43) is shown in Appendix, Proof of (43). Therefore, by applying these conditions, we have

$$u_{ref1} = 2505V, \quad (44)$$
$$u_{ref2} = 3640.3V.$$

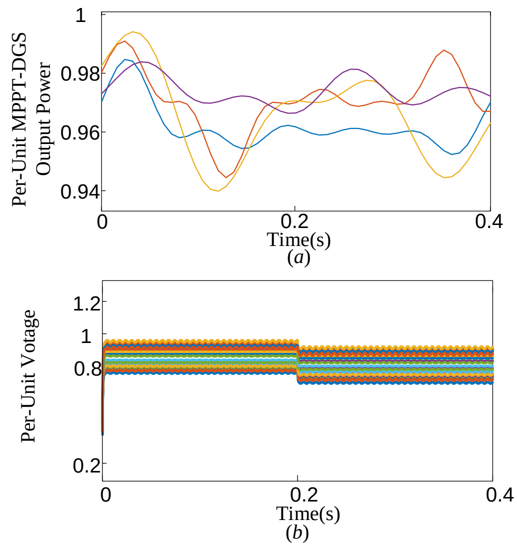

Fig. 7. (a) Output Power of MPPT-DGs. (b) The load Voltages for Case 1.

It is obvious, condition (24) is less conservative than condition (19). Take $u_{ref} = 2505V$, then condition (24) is satisfies while condition (19) is not.

The output voltages of CPLs in case 1 are shown in Fig.7.(b). As shown in Fig.7, when $u_{ref} = 2505V$, the system's equilibrium exists although the powers of renewable generators are time-varying. Therefore, the relevant result verifies the correctness of Theorem 2. Furthermore, for Q1, Theorem 2 is a less conservative condition than the existing result.

B. The Answer to Q2

In this section, one case is designed to make sure that Theorem 3 and Theorem 4 are correct.

The power values of MPPT-DGs are still time-varying and partly random. Assuming that the voltage deviation ε is 0.2. Next, we use case 2 to explain how to use Theorem 3 to regulate the minimum reference voltage of Droop-DGs, which can guarantee that the system has a feasible power-flow solution for a given voltage deviation. Besides, the comparison of Theorem 3 and Theorem 4 is also clarified in this case.

According to Theorem 3 and Lemma 3, it is obvious that the system admits a solution in the convex closed set E if condition (38) holds.

Case 2: The power values of MPPT-DGs are same as case 1, i.e., $P_M(t) \le P_{M,\max}$. The setting values of CPLs change with time as follows

$$\begin{cases} t < 0.2 \\ P_L = [100 \times 1_{10}^{\mathrm{T}}, 150 \times 1_{10}^{\mathrm{T}}, 200 \times 1_{10}^{\mathrm{T}},]^{\mathrm{T}} kW, \\ 0.2 \le t < 0.4 \\ P_L = [180 \times 1_{10}^{\mathrm{T}}, 230 \times 1_{10}^{\mathrm{T}}, 280 \times 1_{10}^{\mathrm{T}},]^{\mathrm{T}} kW, \\ t > 0.6 \\ P_L = [260 \times 1_{10}^{\mathrm{T}}, 310 \times 1_{10}^{\mathrm{T}}, 360 \times 1_{10}^{\mathrm{T}},]^{\mathrm{T}} kW. \end{cases} \quad (45)$$

Next, Theorem 3 and Theorem 4 are both applied to design the reference voltage for case 2 according to Fig.5.

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Define $u_{ref,B}$ represents the minimum value of reference voltage obtained by condition (38), and $u_{ref,C}$ represents the minimum value of reference voltage obtained by condition (39).

By using Theorem 3, $u_{ref,B}$ is obtained by applying condition (38), it yields

$$u_{ref,B} = 2240.5V. \tag{46}$$

By using Theorem 4, since $\varepsilon = 0.2$, by using condition (39), $u_{ref,C}$ is calculated as

$$u_{ref,C} = 2238V. \tag{47}$$

Obviously, condition (39) is less conservative than condition (38).

Take $u_{ref} = 2238V$ for case 2. In order to verify the correctness of Theorem 3 and 4, the load voltages of CPLs need to meet the following range

$$u_{ref}(1-\varepsilon)1_{m+n} \leq u_L \leq u_{ref}(1+\varepsilon)1_{m+n}. \tag{48}$$

In this case,

$$1790.4V \leq u_L \leq 2685.6V. \tag{49}$$

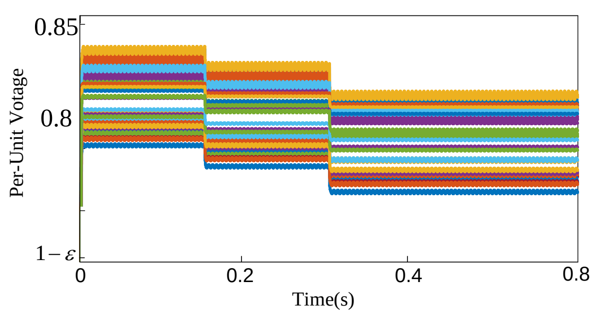

Fig. 8. The load voltages for Case 2.

The results in Fig.8 present that if condition (39) holds, the system remain stable within a given voltage deviation. Thus Theorem 4 is proved.

VI. Conclusion

The existence of the feasible power-flow solution of the DC microgrid considering distributed generations under MPPT Control is analyzed in this paper. At first, an analytical sufficient condition is derived, this condition is less conservative than a existing condition. Next, two sufficient solvability conditions are proposed, which can ensure the existence of the equlibrium within an acceptable voltage deviation. The obtained conditions provide a reference for the establishment of a reliable DC microgrid, whether or not there is a given voltage deviation.

VII. Appendix

A. Proof of (43)

Since $A = Y_{eq}^{-1}\begin{bmatrix} [\![P_L]\!] & \\ & -[\![P_M]\!] \end{bmatrix}$ and $J = u_{ref}^{-2}A$. Then changing (40) into the following form

$$\tau_1 = u_{ref}^{-2}\|A\|_\infty + 3\left(\frac{\|A1_{m+n}\|_\infty}{2}\right)^{\frac{2}{3}} u_{ref}^{-\frac{4}{3}}. \tag{50}$$

$$\tau_2 = 4u_{ref}^{-2}\|A\|_\infty. \tag{51}$$

For (50), we can change it into a cubic equation with one unknown as follows

$$\left(u_{ref}^{\frac{2}{3}}\right)^3 - \frac{3}{\tau_1}\left(\frac{\|A1_{m+n}\|_\infty}{2}\right)^{\frac{2}{3}} u_{ref}^{\frac{2}{3}} - \frac{\|A\|_\infty}{\tau_1} = 0 \tag{52}$$

Applying Cardano formula to solve equation (52), we have

$$\sqrt[3]{u_{ref}^2} = \sqrt[3]{\frac{\|A\|_\infty}{2\tau_1} + \kappa_3} + \sqrt[3]{\frac{\|A\|_\infty}{2\tau_1} - \kappa_3} \tag{53}$$

where

$$\kappa_3 = \sqrt{\frac{\|A\|_\infty{}^2\tau_1 - \|A1_{m+n}\|_\infty{}^2}{4\tau_1^3}}$$

Therefore, we obtain

$$u_{ref} = \sqrt{\frac{\|A\|_\infty}{\tau_1} + 3(\kappa_1 + \kappa_2)} \tag{54}$$

where

$$\kappa_1 = 3\left(\sqrt[3]{\frac{\|A\|_\infty}{2\tau_1} + \kappa_3}\right)^2 \sqrt[3]{\frac{\|A\|_\infty}{2\tau_1} - \kappa_3},$$

$$\kappa_2 = 3\left(\sqrt[3]{\frac{\|A\|_\infty}{2\tau_1} - \kappa_3}\right)^2 \sqrt[3]{\frac{\|A\|_\infty}{2\tau_1} + \kappa_3}.$$

Since we need to ensure that $\tau_1 < 1$, then we can derive u_{ref1} as follows

$$u_{ref1} \geq \sqrt{\frac{\|A\|_\infty}{\tau_1} + 3(\kappa_1 + \kappa_2)}$$

Hence, the first inequality of (43) is proved.

For (51), we can change it into a quadratic equation with one unknown as follows

$$u_{ref}^2 - \frac{4\|A\|_\infty}{\tau_2} = 0 \tag{55}$$

Then, calculating equality (55), we obtain

$$u_{ref} = 2\sqrt{\frac{\|A\|_\infty}{\tau_2}} \tag{56}$$

Since we need to ensure that $\tau_2 < 1$, then we can derive u_{ref2} as follows

$$u_{ref1} \geq 2\sqrt{\frac{\|A\|_\infty}{\tau_2}}$$

Therefore, the second inequality of (43) is proved. Hence, the proof is completed.

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