Randomized Carrier Modulation for Four-Leg Matrix Converter Based on Optimal Markov Chain

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***Abstract* - In order to reject the voltage disturbance of matrix converter's output side from nonlinear loads and improve the comprehensive performance of power converter system, this paper presents a randomized carried modulation strategy based on Markov chain for four-leg matrix converter. It is an optimal randomized carried modulation strategy taking the random transient matrix as variables, and taking the output voltage waveform quality, switching count and electro-magnetic interference level as optimized objectives. An approximate formulation is used due to the complexity of performance indicator's analytical description. This method, on the one hand, is able to reflect the real performance of the system; on the other hand, it simplifies the optimization problem. This modulation strategy has many advantages such as less computation burden and easier realization compared with three-dimensional space vector modulation. Both Simulations and experimental results verify the feasibility of the proposed method.**

***Keyword*: 3-Dimension space vector modulation strategy, four-leg matrix converter, Random carrier modulation strategy, Markov chain**

I. Introduction

Matrix converter[1]-[3] has attracted much attention since 1980s because of its advantages such as bidirectional energy transfer ability, controllable input power factor, compact in structure and small in volume. But all of the merits are based on the preconditions which the power supply and the loads are symmetrical. A novel four-leg matrix converter is presented, it is because the traditional matrix converter with 3×3 matrix structure is incapable of dealing with disturbance from nonlinear loads. In reference [4], a modulation strategy based on space vector modulation is presented, but it requires heavy computation load because of the requirements of coordinate transformation, recognizing of prisms and tetrahedrons, duty ratio calculation based on projection matrix [5]. In order to overcome the shortcomings mentioned above, a new method based on carrier modulation is proposed for four-leg matrix converter in this paper.

Matrix converter, as one important member of power electronics family, like many other power converters, contains a plenty of harmonics component because its operation waveforms are a series of high-speed narrow pulses. In addition, because the matrix system exist parasitic inductance and capacitance, the $\frac{du}{dt}$, $\frac{di}{dt}$ induced by high-speed switches results in serious Electro Magnetic Interference (EMI) problem [6] [9]. In order to meet the requirement of Electro Magnetic Compatibility (EMC), which posed a challenge to design of input or output filter, lots of passive solutions such as making use of laminated bus technique to reduce parasitic inductance, increasing driver resistance and designing proper buffer circuits, can reduce the EMI to a certain extent. But those techniques need extra modification related to peripheral hardware.

Randomized modulation technique can make the discrete spectrum transited into continuous spectrum by introducing random factors to the circuit. So it can mitigate EMI level of switch system effectively, and improve spectrum of energy distribution which concentrate on the switching frequency and its integer multiple frequencies, and reduce acoustic noise and mechanical resonance in adjustable speed drive (ASD) system.

In reference [7], it is very effective to control power spectrum by introducing Markov chain into the modulation procedure of DC/DC converter. In this paper, a Randomized carried modulation based on Markov chain is proposed in terns of the inherent characteristics of matrix converter. The random transient matrix is optimized with the aims of minimizing switching counts and total harmonics distortion

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(THD) of output voltage.

II. CARRIED MODULATION STRATEGY

A. Calculation of offset signal

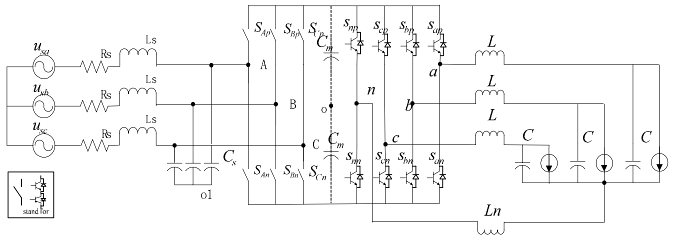

Fig.1. Four-leg matrix converter topology

The basis of carrier based modulation strategy is "voltage second balance" principle. Based on the principle, infinite modulation signals can be selected to synthesize the required output in theory. In three phase system, the degree of freedom of modulation signal is the zero sequence-signal, but for four-leg matrix converter, it is the offset signal. From figure.1, based on Kirchhoff's voltage law, the equation (1), and constraints (2), (3) and (4) will hold.

$$u_{in} = u_{is} + u_{sn} \tag{1}$$

$$-u_{dc} \le u_{in} \le u_{dc} \tag{2}$$

$$-0.5u_{dc} \le u_{no} \le 0.5u_{dc} \tag{3}$$

$$-0.5u_{dc} \le u_{io} \le 0.5u_{dc} \tag{4}$$

Where $i \in \{a,b,c\}$ and point "o" is the potential of virtual midpoint of DC bus, u_{dc} denotes the DC bus voltage. After some manipulations on (1), (3) and (4), a compact inequality which the offset signal should hold is given as

$$-0.5u_{dc} - \min(u_{an},u_{bn},u_{cn}) \le u_{no} \le 0.5u_{dc} - \max(u_{an},u_{bn},u_{cn}) \tag{5}$$

Considering the constraint (2), then inequality (5) can be rewritten in the form of (6).

$$\begin{cases} -0.5u_{dc} \le u_{no} \le 0.5u_{dc} - \max(u_{an},u_{bn},u_{cn}) & when: \max(u_{an},u_{bn},u_{cn}) < 0 \\ -0.5u_{dc} - \min(u_{an},u_{bn},u_{cn}) \le u_{no} \le 0.5u_{dc} & when: \max(u_{an},u_{bn},u_{cn}) < 0 \\ -0.5u_{dc} - \min(u_{an},u_{bn},u_{cn}) \le u_{no} \le 0.5u_{dc} - \max(u_{an},u_{bn},u_{cn}) & othercase \end{cases} \tag{6}$$

$$\begin{cases} \dfrac{-\max(u_{an},u_{bn},u_{cn})}{2}, when \quad \min(u_{an},u_{bn},u_{cn}) > 0 \\ \dfrac{-\min(u_{an},u_{bn},u_{cn})}{2}, when \quad \max(u_{an},u_{bn},u_{cn}) < 0 \\ \dfrac{-\max(u_{an},u_{bn},u_{cn}) - \max(u_{an},u_{bn},u_{cn})}{2}, othercases \end{cases} \tag{7}$$

Inequality (6) represents the range of the offset signal. Different output voltages with different performances in microscopic such as low common voltage, switching loss and distortion, will be achieved by choosing proper offset voltage. In following simulations and experiments, the offset voltage is selected according to (7).

B. Analysis of modulation strategy

Current space vector modulation strategy is carried out at the rectifier side of four-leg matrix converter. Referring to fig.2, the duty ratios are calculated as follows

$$d_\gamma = \sin(k\pi/3 - \theta - \pi/6) \tag{8}$$

$$d_\sigma = \cos(\theta - k\pi/3) \tag{9}$$

Where k represents the section number which the current vector belongs to, and θ denotes the absolute value of phase angle.

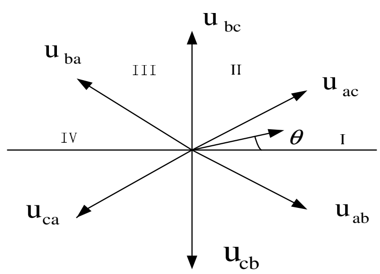

Fig.2. current space vector diagram

In order to make the zero vectors not appear in the modulation of rectifier side explicitly, the normalization operations on duty ratios at rectifier side are used as follows

$$d_{r1} = d_r/(d_r + d_\delta) \qquad (10)$$

$$d_{\delta 1} = d_\delta/(d_r + d_\delta) \qquad (11)$$

Then the local average DC voltage is given as

$$u_{dc} = u_{ab}d_{\gamma 1} + u_{ac}d_{\delta 1} \qquad (12)$$

After being simplified, (12) can be rewritten as

$$u_{dc} = \frac{1.5u_m}{\cos((k-1)\frac{\pi}{3} - \theta)} \qquad (13)$$

Where u_m denotes peak value of grid phase voltage. As can be noted in (13), the local DC voltage fluctuates with the frequency of six time fundamental frequency of power supply. As shown in figure.2, the reference current vector lie in section I, where DC voltage is synthesized by voltage u_{ab} and u_{ac} .It is assumed that there exists a real DC voltage u_{dc} , and the required output voltage is u_{io}^* ,then the normalized modulation signal is denoted as $\bar{u}_{io} = 2u_{io}^*/u_{dc}$ .However, the DC voltage in the middle of matrix converter is synthesized by u_{ab} and u_{ac} . If we just focus on inverter terminal of matrix converter, modulation signals in respective sub-carrier cycle have many choices. But considering the restraints from rectifier, the only choice is that each normalized modulation signals should be equal to $\bar{u}_{io}$.

C. carrier and pulse synchronization

The flexibility of space vector synthesis strategy lies in the position of zero vectors. While the corresponding degree of freedom of carrier modulation lies in carrier waves' shapes. Taking the carriers in the shape of isosceles-triangle and right-angled triangle for example, we will get eight different combinations showed in figure.3 according to positions and shapes of carriers. Class A, B, G and H have the advantages of low harmonics in output voltage due to the symmetrical carrier waves, while the others possess of the merit of less switching count in per carrier cycle. Because zero vectors in any case are placed in which the commutation in rectifier side occurs; the advantage of zero current commutation can be kept in this strategy as in [3].

Matrix converter is different from front-end PWM converter with almost the same topology. This is because matrix converter's modulation strategy has a prominent characteristic that the decoupling between rectifier and inverter is based on strict synchronization. Instead voltage source front-end PWM converter establishes the decoupling by a great electrolytic capacitor. So the synchronization is the key to normal operation of matrix converter. A class of classic synchronization switching sequence is shown in figure.4.

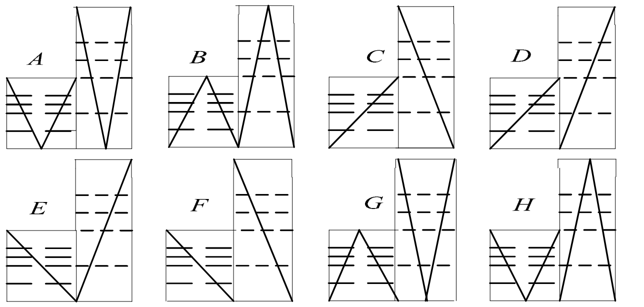

Fig.3 several carrier wave forms

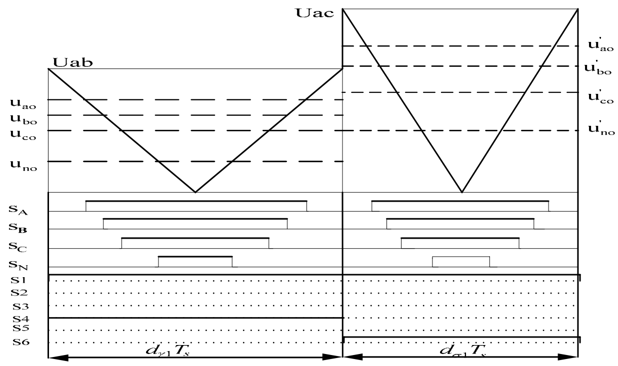

Fig.4 Switch synchronization diagram based on carried modulation

III. MARKOV-CHAIN BASED CARRIER MODULATION

Switching different carriers randomly is also a means to introduce random mechanism. There are eight carriers which are available to synthesize the required output voltage in figure.3. Six of them are selected to serve as states of Markov chain. They will be divided into three groups according to their unique features. Group I includes carrier A and E, featured by beginning and ending the modulation procedure with zero vector $(1,1,1,1)$, Group II includes carrier B and C, featured by beginning and ending the procedure with the other zero vector $(0,0,0,0)$, Group III consists of carrier G and H, characterized by beginning with a zero vector and ending with the other. Such combinations above have the advantage that the carrier in Group I or II can switch freely with minimum switching count in respective group. A proper state in Group III will serve as a transient state if one carrier will transit from Group I to Group II. Considering the constraints such as minimum switching count and low THD, a Markov chain relied by proposed Stochastic Processes is depicted in figure.5. where the state A,B,C,E,H stands for the carrier A,B,C,E,H in figure.4 respectively. In order to explain the stochastic Processes, a transient random matrix is defined as follows.

$$p=\begin{bmatrix} P_{AA} & 0 & 0 & P_{AE} & 0 & P_{AH} \\ 0 & P_{BB} & P_{BC} & 0 & P_{BG} & 0 \\ 0 & P_{CB} & P_{CC} & 0 & P_{CG} & 0 \\ P_{EA} & 0 & 0 & P_{EE} & 0 & P_{EH} \\ P_{GA} & 0 & 0 & P_{GE} & 0 & P_{GH} \\ 0 & P_{HB} & P_{HC} & 0 & P_{HG} & 0 \end{bmatrix} \quad (14)$$

Where A,B,C,D,E,H stands for state 1,2,3,4,5,6 respectively. The (i,j) entry stands for the probability for the state transiting from the ith state to the jth state, and the sum of entries in any row is 1.

In order to evaluate performance of the proposed modulation strategy, the steady distribution vector π is defined as follows.

$$\pi=[\pi_1 \ \ \pi_2 \ \ \pi_3 \ \ \pi_4 \ \ \pi_5 \ \ \pi_6] \quad (15)$$

$$\pi=\pi\cdot p \quad (16)$$

$$\pi_1+\pi_2+\pi_3+\pi_4+\pi_5+\pi_6=1 \quad (17)$$

Where any element in the vector π stands for the percent of average time which corresponding state will experience in steady state. In addition, the vector π has to satisfy the constraint (16) and (17).

Because the system introduces random mechanism, all related performance indexes must be weighted by Statistical properties. To get a kind of modulation with comprehensive optimal performance, a multi-objective optimization problem will be formulated.

The optimization problem can be solved in two steps. Step I, solving optimal steady distribution vector; step II, solving optimal transient random matrix.

Firstly, in order to get the optimal steady distribution, an objective function is defined as

$$J=\lambda_1\times E(Loss)+\lambda_2\times E(THD) \quad (18)$$

$$\lambda_1+\lambda_2=1 \quad (19)$$

Where λ is the weight; $E(Loss)$ is the mathematical expectation of switching count, and $E(THD)$ denotes expectation of THD of output line-to-line voltage.

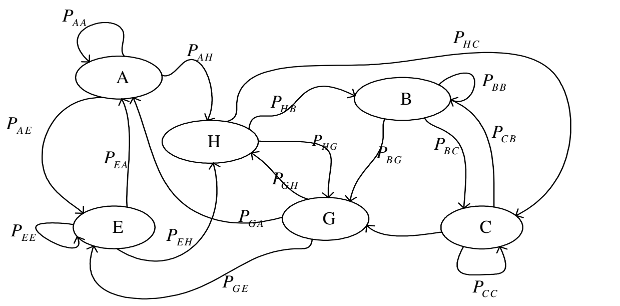

Fig.5 Markov chain

According to (7), it will be noted that the system works at continuous PWM state. Denoting sampling frequency as f_c, then switching frequency for carrier A, B is $4f_c$, $2f_c$ for carrier C, E, and $5f_c$ for carrier G, H. So the expectation of switching count is given as.

$$E(loss)=f_c\cdot(4\pi_1+4\pi_2+2\pi_3+2\pi_4+5\pi_5+5\pi_6) \quad (20)$$

As so far, output voltage waveform quality can be evaluated in many ways such as flux harmonics distortion [9] and total harmonic distortion. Flux harmonics distortion has the advantage of simplicity and easy to describe in analytical expression. But in this case, waveform quality is related to power supply and 3-dimention output voltage vector, the complete analytical expression about flux harmonics distortion will be too complicated. In this study, an approximated approach is chosen to formulate waveform quality for random modulation strategy.

$$\begin{aligned} E(THD)&=\pi_1\cdot THD_A+\pi_2\cdot THD_B+\pi_3\cdot THD_C \\ &+\pi_4\cdot THD_E+\pi_5\cdot THD_G+\pi_6\cdot THD_H \end{aligned} \quad (21)$$

Where $THD_i \ i\in\{A,B,C,E,G,H\}$ stand for the THD under carrier i, it can be got via simulations.

Substituting (20) and (21) into (18), it is easy to get the optimal vector π.

Then, according to the normalized optimal vector π^* solved above, the second objective function is given by

$$J=(line(\lim_{m\to\infty} p^m)-\pi^*)^2 \quad (22)$$

$$\sum_{j=1}^{6} p^{(i,j)}=1 \quad (23)$$

$$p^{(i,j)}>0 \quad (24)$$

Where return value of the function $line()$ is an arbitrary row vector in a matrix. In fact, $line(\lim_{m\to\infty} p^m)$ is equal to the steady distribution vector π. In the practical application, m in (22) being set to 20 will be well in this case. Because the complexity of the problem, a simple Genetic Algorithm is used to solve the transient random matrix; and the real coding is used here. Because of the limitation of space, the detailed procedures about GA are omitted here.

IV. Simulation and Experiment

To verify the validity and feasibility of the proposed modulation strategy, Matlab/simulink is used to simulate the four-leg matrix converter system. Main parameters for simulation are listed in Tab.1.

Tab.1 parameter list

device	parameters	
Power supply	V_s (RMS) $/V$	220
	$\omega_i/(rad\cdot s^{-1})$	100π
Sampling frequency	$f(HZ)/s$	2500
Input filter	R_s/Ω	1
	L_s/mH	0.2
	$C_s/\mu F$	30

Output line-to-line Voltage's power spectrums under strategy-A, C, and random modulation strategies are depicted in fig.6 (a), fig.6 (b) and fig.6(c) respectively. It is easy to draw a conclusion that the harmonic components under strategy-C at integer multiples of switching frequency is larger than that under strategy-A resulting from symmetry of local PWM pulses. Another evident conclusion is that power spectrum transit from discrete spectrum to continuous spectrum due to the introduction of random mechanism.

A four-leg matrix converter prototype rated at 3.7Kw is developed in the laboratory. Its controller is composed of a DSP (TMS2407) and CPLD (EPM1270). The DSP is responsible for sampling voltages, calculating duty ratios, synthesizing modulation signals and generating random numbers. The CPLD serve as a PWM generator for matrix converter. The schemes of experimental prototype are: symmetrical three-phase power supply with line-to-neutral voltage 120 volts RMS are fed into matrix converter, voltage transfer ratio is set to be 0.75 and the prototype operates in the condition of no load. The parameters about Input filter are the same as that listed in Tab.1. Output line-to-line voltage with the frequency 25HZ is showed in fig.7. Fast Fourier Transform (FFT) analysis results under randomized modulation and strategy-A in the range from 0 to 12.5 kHz are showed in fig.8 (a) and fig.8 (b) respectively. The experiment results indicate that randomized modulation smoothes the harmonics greatly at the frequency of 2.5, 5, 7.5kHz etc. Meanwhile the experimental results showed in fig.9 represent the harmonics status of two different modulation strategies in a more wide range. The experimental results about them coincide with simulation results shown in fig.6.

V. Conclusion

The emergence of four-leg matrix converter has broadened area of application of the matrix converter. The proposed carrier modulation strategy based on optimal Markov chain improves efficiency of the system, EMC, and THD. The modulation is simple in realization and comprehension for engineers, and easy to be extended to other types of power converter's applications.

Acknowledgement

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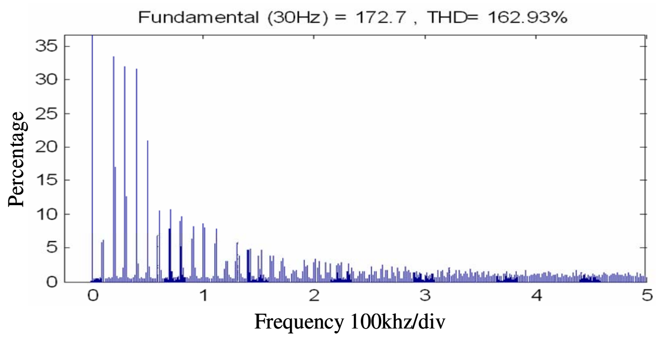
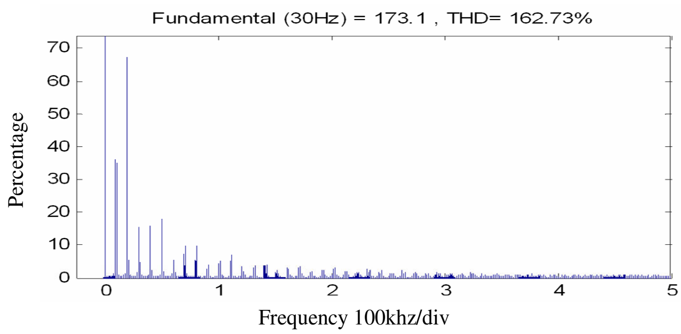
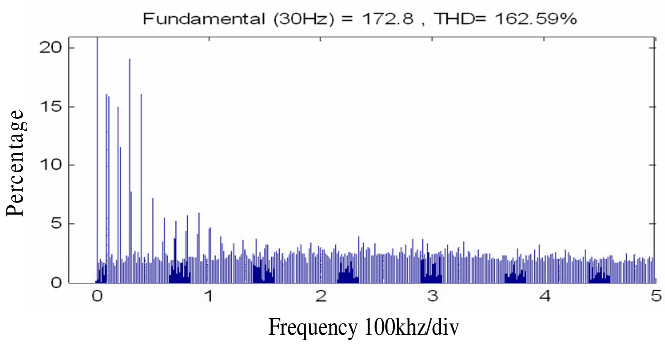


(a) FFT analysis under carrier A



(b) FFT analysis under carrier C



(c) FFT analysis for randomized carrier

Fig.6 FFT analysis of output line-to-line voltage

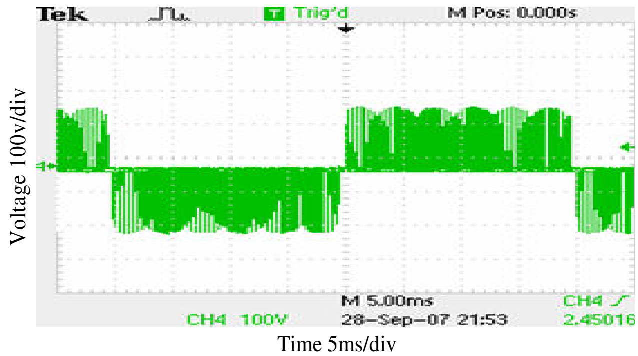
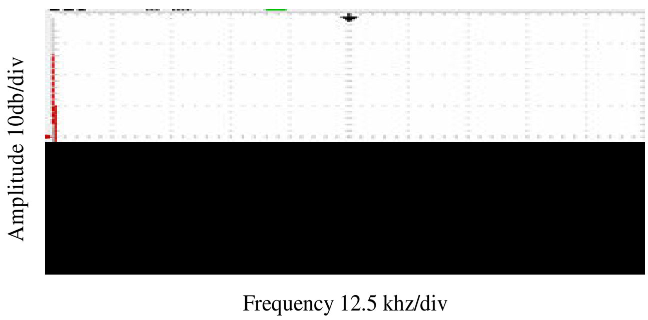
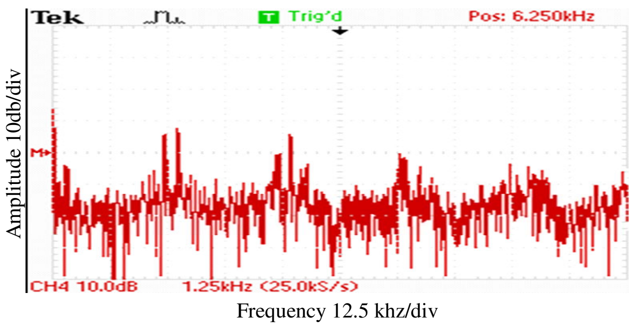


Fig.7 output line-to-line waveform



(a) FFT analysis for randomized carrier



(b) FFT analysis under carrier-A

Fig.8 FFT analysis in the range from 0 to 12.5KHZ

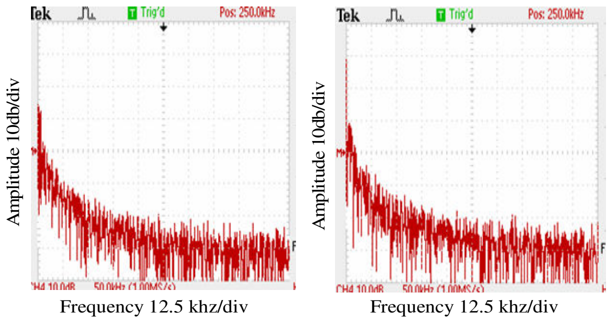


Fig.9 FFT analysis under Carrier Modulation A (left) and FFT analysis under random Carrier Modulation in the range from 0 to 125KHZ

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