A Matrix Converter Modulation based on Mathematical Construction

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***ABSTRACT*: Various modulation schemes have been proposed for its control. But the most control methodology of matrix converter operation is too complex. In this paper, a simple modulation based on mathematical construction is proposed. This modulation does not need any sector information. It is simple and easy to comprehend and implement. It can ensure maximum input power voltage transmission ratio of 0.866 and controllable input power factor. Theoretical considerations are supported by experimental results.**

***KEY WORDS*: matrix converter; mathematical construction; modulation**

I. INTRODUCTION

Matrix converter (MC) is a direct AC/AC power conversion device with several advantages compared to the traditional pulse width modulation (PWM) inverter such as bidirectional energy flow, sinusoidal input and output current, as well as no large energy storage components. After the initial appearance of Matrix Converter in 1976, several modulation methods have been proposed, including Direct Transfer Method [1], Indirect Space Vector Modulation [2], Direct Space Vector Modulation [3] and Carrier Based Modulation [6]. However, the above modulation methods are complicated when implemented in microcontrollers. This work presents a novel modulation method based on mathematical construction, which belongs to carrier modulation in essence but is different from the previous result[6] in concept. This method is very instructive for students majoring in power system and power electronics. Meanwhile it is easy to understand and simple for microcontroller implementation. Both simulation and experiment results verify the effectiveness of the proposed method.

II. SCHEME AND PRINCIPLE OF MATRIX CONVERTER

A. Topology of Matrix Converter

Matrix converter, as showed in Fig.1 (S_{jk} ,i＝A,B,C; j=a,b,c) are ideal switches), contains 9 bidirectional switches. It is possible to control these 9 switches to synthesis the reference output voltage.

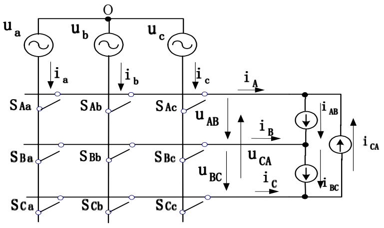

Fig.1 The simplified topology of matrix converter

B. Operation Principle of Matrix Converter

Matrix converter can be considered to be a six-port network, with three input ports and three output ports. By properly control the nine bidirectional switches, reference output voltage can be synthesized from input voltage. Let input phase voltage and output current be:

$$\begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} = U_{im} \cdot \begin{bmatrix} \cos(\omega_i t) \\ \cos(\omega_i t - 2\pi/3) \\ \cos(\omega_i t + 2\pi/3) \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = I_{om} \cdot \begin{bmatrix} \cos(\omega_o t + \varphi) \\ \cos(\omega_o t + \varphi - 2\pi/3) \\ \cos(\omega_o t + \varphi + 2\pi/3) \end{bmatrix} \tag{2}$$

The switching configuration can be represented by a Modulation Matrix M . Therefore, the three phase output voltages are:

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$$\begin{bmatrix} u_A \\ u_B \\ u_C \end{bmatrix} = M \cdot \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} \qquad (3)$$

where, $M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$,

m_{ij}（i=1,2,3; j=1,2,3）corresponds to the on-time ratio of each bidirectional switch. Several constrains should be met to keep the physical availability: (1) m_{ij} must be greater or equal to zero (ref Equ 4); (2) input voltage cannot be shorted and output current cannot be open (ref Equ 5-7).

$$0 \le m_{ij} \qquad (4)$$

$$m_{11} + m_{12} + m_{13} = 1 \qquad (5)$$

$$m_{21} + m_{22} + m_{23} = 1 \qquad (6)$$

$$m_{31} + m_{32} + m_{33} = 1 \qquad (7)$$

The modulation strategy of Matrix Converter is to find a modulation matrix M, which would determines the performance of Matrix Converter.

III A Simple Modulation Strategy Of Matrix Converter

From the above analysis, in order to control arbitrary output voltage (that is not related to the input frequency and voltage), one feasible solution is indirect SVPWM [2]. Theoretically, input voltage is rectified virtual DC voltage, which is later converted to the reference output voltage. An intermediate matrix can be constructed by equation (8).

$$M" = \begin{bmatrix} m_{11}" & m_{12}" & m_{13}" \\ m_{21}" & m_{22}" & m_{23}" \\ m_{31}" & m_{32}" & m_{33}" \end{bmatrix} = K(\omega_o) \cdot D^T(\omega_i) =$$

$$\begin{bmatrix} K_a \\ K_b \\ K_c \end{bmatrix} \cdot \begin{bmatrix} \cos(\omega_i t - \beta) \\ \cos(\omega_i t - 2\pi/3 - \beta) \\ \cos(\omega_i t + 2\pi/3 - \beta) \end{bmatrix}^T =$$

$$\begin{bmatrix} K_a\cos(\omega_i t - \beta) & K_a\cos(\omega_i t - \beta - 2\pi/3) & K_a\cos(\omega_i t - \beta + 2\pi/3) \\ K_b\cos(\omega_i t - \beta) & K_b\cos(\omega_i t - \beta - 2\pi/3) & K_b\cos(\omega_i t - \beta + 2\pi/3) \\ K_c\cos(\omega_i t - \beta) & K_c\cos(\omega_i t - \beta - 2\pi/3) & K_c\cos(\omega_i t - \beta + 2\pi/3) \end{bmatrix}$$

(8)

where $D^T(\omega_i) = \begin{bmatrix} \cos(\omega_i t - \beta) \\ \cos(\omega_i t - 2\pi/3 - \beta) \\ \cos(\omega_i t + 2\pi/3 - \beta) \end{bmatrix}^T$, corresponds to the rectifier stage and $K(\omega_o) = \begin{bmatrix} K_a \\ K_b \\ K_c \end{bmatrix}$, corresponds to the rectifier stage.

$K_a = K\cos(\omega_o t)$；

$K_b = K\cos(\omega_o t - 2\pi/3)$；

$K_c = K\cos(\omega_o t + 2\pi/3)$

Replace M by $M"$ in Equation（2）, so the A phase output voltage can be represented as:

$$u_A = 1.5KU_{im}\cos(\beta)\cdot\cos(\omega_o t) \qquad （9）$$

From equation (9), the output frequency is not related to input frequency. But matrix $M"$ can not satisfy the constraints set by equation (4)~(7). Therefore, further mathematical construction is needed.

It can be found that the sum of each row in $M"$ is zero. So offset values x, y, z are added to each column of $M"$, as showed in Equation (10). The added values in modulation matrix only change the line-to-phase output voltages, while the line-to-line output voltages remain the same.

$$M' = \begin{bmatrix} m_{11}' & m_{12}' & m_{13}' \\ m_{21}' & m_{22}' & m_{23}' \\ m_{31}' & m_{32}' & m_{33}' \end{bmatrix} = \begin{bmatrix} m_{11}"+x & m_{12}"+y & m_{13}"+z \\ m_{21}"+x & m_{22}"+y & m_{23}"+z \\ m_{31}"+x & m_{32}"+y & m_{33}"+z \end{bmatrix} \qquad （10）$$

Take the first column for example, x should be found to meet Equation (11)~(13).

$$m_{11}' = m_{11}" + x \ge 0 \qquad （11）$$

$$m_{21}' = m_{21}" + x \ge 0 \qquad （12）$$

$$m_{31}' = m_{31}" + x \ge 0 \qquad （13）$$

Obviously, $x \ge -\min(m_{11}", m_{21}", m_{31}")$. Similar method can be used for the rest two columns, leading to the following in equation (14).

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$$\begin{cases} x \ge -\min(m_{11}'', m_{21}'', m_{31}'') \\ y \ge -\min(m_{12}'', m_{22}'', m_{32}'') \\ z \ge -\min(m_{13}'', m_{23}'', m_{33}'') \end{cases} \qquad (14)$$

To meet the constraints set by Equation (5), (6) and (7), one obvious method is to add an offset value D to every elements of matrix M'. D can be expressed as follows,

$$D = \frac{1-(x+y+z)}{3}, \; (D \ge 0) \qquad (15)$$

Let

$$m_{ij} = m_{ij}' + D \quad (i=1,2,3;j=1,2,3) \qquad (16)$$

Now, the construction of modulation M is completed. After some manipulations, the mathematic expressions of the input currents and output line to line voltages can be stated as follows respectively.

$$\begin{cases} i_a = 1.5KI_{om}\cos(\varphi)\cdot\cos(\omega_i t + \beta) \\ i_b = 1.5KI_{om}\cos(\varphi)\cdot\cos(\omega_i t - 2\pi/3 + \beta) \\ i_c = 1.5KI_{om}\cos(\varphi)\cdot\cos(\omega_i t + 2\pi/3 + \beta) \end{cases} \qquad (17)$$

$$\begin{cases} u_{AB} = 1.5KU_{im}\sqrt{3}\cos(\beta)\cdot\cos(\omega_o t + \pi/6) \\ u_{BC} = 1.5KU_{im}\sqrt{3}\cos(\beta)\cdot\cos(\omega_o t - \pi/2) \\ u_{CA} = 1.5KU_{im}\sqrt{3}\cos(\beta)\cdot\cos(\omega_o t + 5\pi/6) \end{cases} \qquad (18)$$

The proposed modulation method can be summarized as follows: (1) the sampled input phase voltage, along with reference output voltage, is used to calculate the intermediate matrix M'' (equ 8); (2)According to equ (14)-(16), proper offset values are added to the intermediate matrix M'', leading to the required modulation matrix M.

From (14), it can found that x, y and z are arbitarily selected, which means different selections can lead to different modulation performance. The simplest method, boundary value, is used in this work (Equ. 19).

$$\begin{cases} x = -\min(m_{11}'', m_{21}'', m_{31}'') \\ y = -\min(m_{12}'', m_{22}'', m_{32}'') \\ z = -\min(m_{13}'', m_{23}'', m_{33}'') \end{cases} \qquad (19)$$

Using this method, the maximum voltage transfer ratio is 0.866, which is prooved as follows.

According to the input/output sector determination method in [3], input sector and output sector are not related. Therefore, without losing generality, assume the input voltages are $u_a>u_b>0>u_c$; and the output volltages are $U_a>U_b>U_c$. From (8), the smallest value in the first column is:

$$K_C\cos(\omega_i t - \beta)$$

The smallest value in the second column is:

$$K_C\cos(\omega_i t - 2\pi/3 - \beta),$$

the smallest value in the third column is:

$$K_a\cos(\omega_i t + 2\pi/3 - \beta)$$

From (16), it can be found that: add 1 to both side of the equation:

$$1-(x+y+z) = 1+\cos(\omega_i t + 2\pi/3 - \beta)\cdot(K_a - K_c)$$

$$\ge 1 - K\cdot\sqrt{3}\cdot\cos(\omega_o t)$$

From the constrain in (14),

$$1-(x+y+z) > 1 - K\cdot\sqrt{3}\cdot\cos(\omega_o t) \ge 0$$

So the parameter K should meet $K \le \sqrt{3}/3$. Substitute k in Equ (9):

$$U_A \le 0.866U_{im}\cos(\beta)\cos(\omega_o t)$$

which means the voltage transfer ratio is equal or smaller than 0.866. The situations in other sectors can be verified in a similar way.

IV. SIMULATION AND EXPERIEMNT RESULTS

A simulation is carried out under MATLAB 6.5. The input voltage is $220V_{rms}$/50Hz. Input filter parameters are Lf=1mH, Cf=10μF. Reference output frequency is set to 25Hz. The simulation results about output line-to-line voltage and output current are shown in Fig 2.

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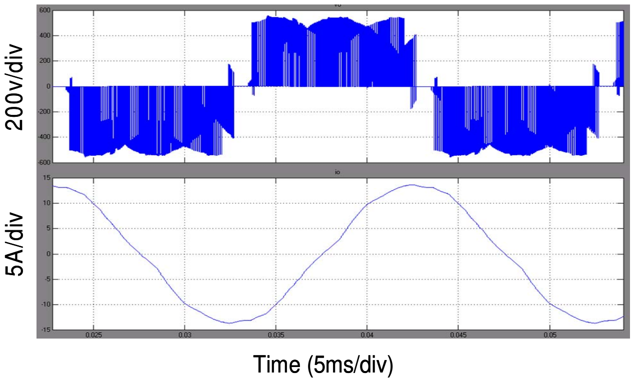


Fig.2 Simulation results under the proposed method

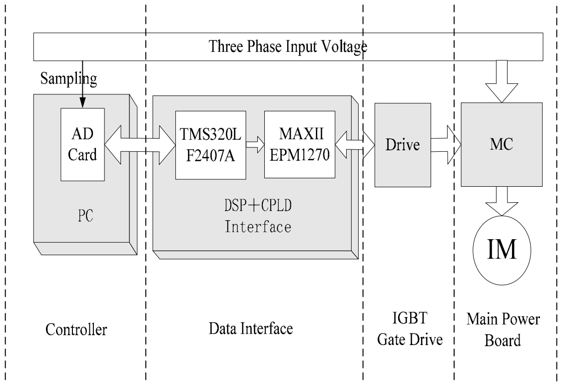
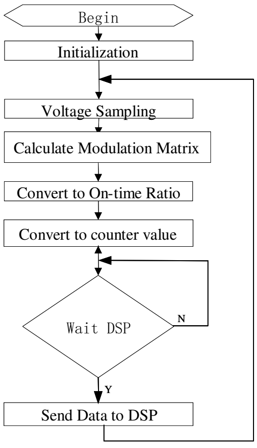


Fig.3 Block diagram of the matrix converter control system

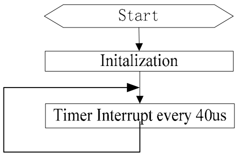
Matrix converter experimental system with modular design thinking, overall structure is divided into three modules shown in the gray box in figure 3, the overall structure of the system as shown in figure 3. in order to implement the control of AC induction motor, at first, the three-phase power source's amplitude, phase and the direction of output current should be detected, according to the above method, the modulation matrix M can be calculated, then these duty ratios in matrix M are converted into the time in which switches are on and off.

The proposed control system is implemented in a AD-card plus DSP development board. The data calculated are sent to CPLD by AD-card, through DSP. In CPLD, a finite state machine is design to implement the 4-step commutation algorithm and the symmetry PWM pattern [3]. The flow charts of PC and DSPare shown in Fig 4.

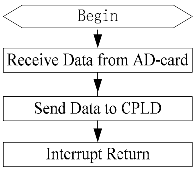
The scheme of Matrix Converter prototype is showed in Figure 5. Input voltage is 100Vrms. Input filter parameters[4-5] are L=0.5mH and C=10uF. Switching frequence is set to 5kHz. The load, a induction motor, is rated at 750W. When output voltage frequence is set to 25Hz, the experiment results are showed in Figure 6. It is demonstrated that the input current is almost sinusoidal; the frequency of output voltage is equal to its reference value; and low-order harmonics in input current and output current are low.



(a) Flow chart of PC



(b) DSP Main Program



(c) DSP ISR

Fig 4 Flow charts of the PC and DSP

V. CONCLUSIONS

This work presents a mathematical structure based modulation strategy for matrix converter. The modulation strategy does not require sector determination, which is simple and easy to understand. The novel modulation strategy presented in this paper has the potential to be used in other power conversion devices.

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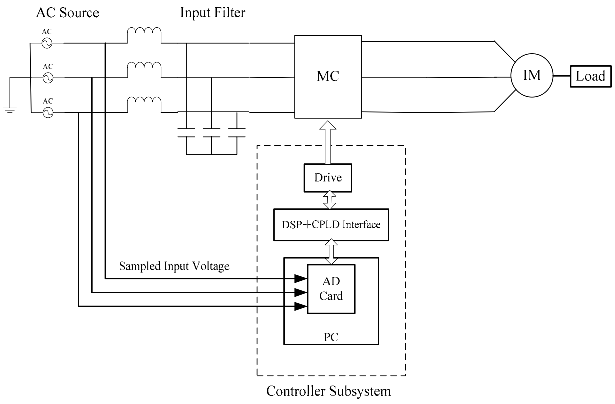
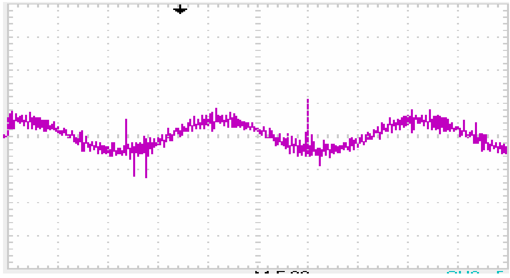
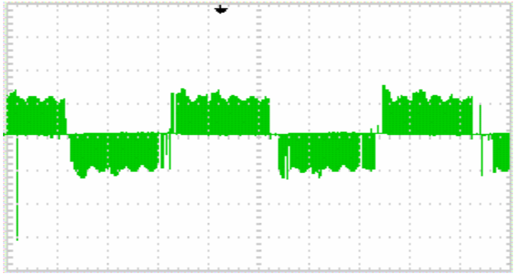


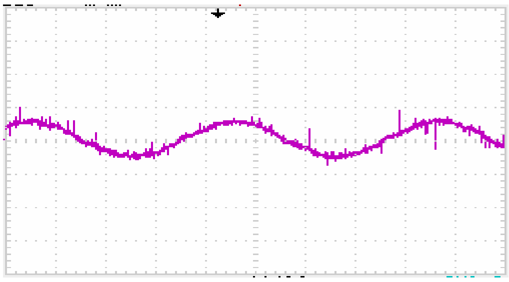
Fig.5 Experimental configuration



(a) Input Current(1A/div)， (5mS/div)



(b) Output Line-to-Line voltage (100V/div), (10mS/div)



(c)Output Current (1A/div), Time(10mS/div)

Fig.6 Waveform of the matrix converter

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