A Decentralized Control with Unique Equilibrium Point for Cascaded-type Microgrid

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Abstract—The existing reactive power polarity-dependent decentralized control for cascaded-type microgrid (CMG) has the problem of multiple equilibrium points. As a result, some undesired operating states may occur. To address this problem, we propose a new decentralized power sharing control scheme. The uniqueness of equilibrium point and its small signal stability are proved. Finally, the feasibility of the proposed method is verified by simulation and experiment.

Index Terms—Cascaded micro-converters, cascaded-type microgrid, decentralized control.

I. INTRODUCTION

Cascaded H-bridge micro-converters are widely used in integrating distributed generations (DGs) and loads to form a cascaded-type microgrid (CMG) [1-5]. In the grid-connected CMG, the power balance control methods have been intensively studied [1-2]. Literature [6] proposes a control scheme for controlling the active and reactive power of grid-tied ac-stacked photovoltaic inverter by single member phase compensation, which is a solution of high reliability and low-cost. Further, a fully decentralized power balance control is presented in [7]. However, these methods cannot be directly used to the islanded operation mode for the CMG. In the islanded mode, [3] firstly introduced a CMG, and an inverted power factor droop control was presented under the resistive-inductive (RL) loads. Further, [4] presented an f-P/Q method to broaden the application scope of [3] when feeding the RL and resistive-capacitive (RC) loads. However, the equilibrium point of [4] is related to the initial state of system. The undesired equilibrium points may lead to the serious load voltage-sag and undermine the power sharing control.

To overcome the limitation of [4], this letter proposes a new decentralized power sharing control in the CMG. Its main features are summarized as follows: 1) the control scheme only needs the local information; 2) the CMG always holds an unique equilibrium point regardless of the initial state of system; 3) frequency synchronization and power sharing control can be realized autonomously under both RL and RC loads.

II. ANALYSIS OF PROPOSED CONTROL STRATEGY

A. Equivalent Models of CMG

Fig. 1 shows the configuration of an islanded CMG consisting of \( n \) DG units. The output active power \( P_i \) and reactive power \( Q_i \) of the \( i^{th} \) DG are derived as follows

\[
P_i + jQ_i = V_i e^{j\theta_i} \left( \sum_{j=1}^{n} V_j e^{j\theta_j} / |Z_{load}| e^{j\theta_{load}} \right)^* \tag{1}
\]

where \( V_i \) and \( \theta_i \) represent the voltage and phase angle of the \( i^{th} \) DG unit. The subscript \( j \) is the serial number of the \( j^{th} \) DG unit. \( Z_{load} \) and \( \theta_{load} \) are the impedance and impedance angle of the generalized load which includes transmission line and the load, respectively. Then, the power transmission characteristics of the \( i^{th} \) DG are derived as follows

\[
P_i = V_i \sum_{j=1}^{n} V_j \cos(\delta_i - \delta_j + \theta_{load}) \left| Z_{load} \right| \tag{2}
\]

\[
Q_i = V_i \sum_{j=1}^{n} V_j \sin(\delta_i - \delta_j + \theta_{load}) \left| Z_{load} \right| \tag{3}
\]

B. Proposed Control Strategy

Assume that all the DGs have the same capacity. The proposed decentralized control strategy of the CMG is expressed as

\[
o_\omega = o^* + m \text{sgn}(Q_i) P_i \tag{4}
\]

where \( o_i \) is the angular frequency. \( o^* \), \( V^* \) is the nominal angular frequency and voltage amplitude. \( \text{sgn}(\cdot) \) is a signum function. \( m \) is a positive coefficient. \( \bar{V}_i \) is the voltage vector. Equation (5) can be rewritten as

\[
\bar{V}_i = V^* \text{sgn}(P_i) \tag{6}
\]

Clearly, the proposed scheme in (4) and (5) only needs the local information of each DG, thus the decentralized manner is realized.

C. Steady-state Analysis

When the CMG is in the steady-state, from (4), we have

\[
\text{sgn}(Q_i) P_i = \text{sgn}(Q_j) P_j, i \neq j \tag{7}
\]

where \( i, j \in \{1, 2, \ldots, n\} \). According to (7), there are two types of steady-state operation states.

\[
\begin{align*}
&1) Q_i = Q_j, \text{or} (II) Q_i = -Q_j \quad (P_i = P_j, P_i = -P_j) \tag{8}
\end{align*}
\]

According to (8), the vector diagram is shown in Fig. 2.
Undesired is the power sharing coefficient of the \( \omega \). Since \( \hat{\delta}_i = \omega \), combining (2), (4) and (5), then yields
\[
\hat{\delta}_i = \omega^* + m \text{sgn}(q) \left( V^* \right)^2 \sum_{j=1}^{n} \text{sgn}(p_j) \text{sgn}(p_i) \cos(\delta_j - \delta_i + \delta_{\text{load}}) \left| Z'_{\text{load}} \right|
\]
Linearization (15) around the equilibrium point, we have
\[
\Delta \hat{\delta}_i = -m \text{sgn}(Q_{\text{load}}/n) \left( V^* \right)^2 \sin(\delta_{\text{load}}) \sum_{j=1}^{n} (\Delta \delta_j - \Delta \delta_i) \left| Z'_{\text{load}} \right|
\]
Rewrite (16) in the matrix form
\[
\dot{X} = AX
\]
where \( X = \begin{bmatrix} \Delta \delta_1 & \cdots & \Delta \delta_n \end{bmatrix}^T \), \( A = K(nI_{m \times n} - I_n I_n^T) \).
And the eigenvalues of the system matrix \( A \) are expressed as follows
\[
\lambda_1(A) = 0, \lambda_2(A) = \cdots = \lambda_n(A) = nK
\]
The zero eigenvalue corresponds to rotational invariance of the dynamics, as depicted in [8]. In case of the RL or RC loads, \( K < 0 \). Thus, the system is stable.
In practice, the capacities of different DGs are usually unequal.
Assume that the rated capacity of the \( i^{th} \) DG unit is \( S_i \). For power sharing under this circumstance, we can modify (6) as
\[
V_i = \frac{S_i}{S_{\text{sum}}} V_{\text{sum}} \text{sgn}(p_i)
\]
where \( V_{\text{sum}} \) is the total nominal voltage amplitude of the CMG, and \( S_{\text{sum}} \) is the total capacity of the CMG.
In addition, for the different ratios of active power sharing, we can modify (4) as
\[
\text{sgn}(q) \sum_{j=1}^{n} \text{sgn}(p_j) \left| Z'_{\text{load}} \right|
\]
where \( m_i \) is the power sharing coefficient of the \( i^{th} \) DG. In the steady state, for the \( i^{th} \) and \( j^{th} \) DG \( P_i : P_j = m_i : m_j \) will be obtained.

III. SIMULATION AND EXPERIMENTAL RESULTS

A. Simulation Results

Fig. 3. Simulation results.
The simulation parameters of the system comprised of four DGs are listed in Table I. The initial phase angle $(\delta_1, \delta_2, \delta_3, \delta_4)$ is set as $(\pi/3, -\pi/3, 0, 0)$. 

**Case 1:** The f-P/Q method in [4] is performed, the power sharing results are shown in Fig.3(a.1) and (b.1). The output active power of DG2 is negative (the second type of equilibrium point in (8)). The frequency waveforms are shown in Fig.3(c.1), and the serious load voltage-sag occurs as depicted in Fig.3(d.1). 

**Case 2:** The proposed strategy is adopted in this case. The power allocations of all DGs are desired shown in Fig.3(a.2) and (b.2). The frequencies are depicted in Fig.3(c.2), which verifies that the frequency synchronization without communications is obtained. The load voltage amplitudes are shown in Fig.3(d.2), in which the simulation value is under the set value slightly due to line voltage drop. As seen, the proposed method can achieve the accurate power sharing and obtain satisfying load voltage quality. 

**Case 3:** The simulation case where the DGs have different capacities is performed. In this case, set $S_1:S_2:S_3:S_4 = 5:4:4:4$, $V_{\text{sum}}^* = 311 \text{V}$. The obtained active and reactive power are shown in Fig.3(a.3) and (b.3), respectively. The frequencies are shown in Fig.3(c.3), and the load voltage amplitude is depicted in Fig.3(d.3). The simulation results indicate that the proposed scheme can realize the frequency synchronization and active power sharing autonomously under the different capacities of DGs. 

**Case 4:** The case with the different ratios of active power sharing is performed, where $m_1:m_2:m_3:m_4 = 4:5:4:5$ is set. The active power allocations are shown in Fig.3(a.4), where $P_1:P_2:P_3 = 5:4:4:4$ is realized. The waveforms of reactive power sharing, frequency and the load voltage amplitude are shown in Fig.3(b.4), (c.4) and (d.4), respectively. This results indicate that the different ratios of active power sharing is realized under the proposed method.

**B. Experimental Results**

A CMG comprised of two DGs is built, and the proposed scheme is carried out. The experimental parameters are list in Table II. The experimental waveforms are shown in Fig.4, in which a temporary zero-current is caused by the nature switching time of the physical switch. From experimental results shown in Fig.5, the system can share active and reactive power accurately under RL and RC loads, and obtain a satisfying transient response.

### Table I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
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<tr>
<td>$V^*$ (V)</td>
<td>311/4</td>
<td>$Z_{\text{line}}$ (Ω)</td>
<td>0.1+j0.314</td>
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<tr>
<td>$f'/f$ (Hz)</td>
<td>50/[49, 51]</td>
<td>$Z_{\text{load}}$ (Ω)</td>
<td>10.4+j3.8 in [0, 5]s</td>
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<tr>
<td>$m$ (rad/sec.w)</td>
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<td>8.068-j3.978 in [1, 2]s</td>
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</table>

This letter reports a new decentralized power sharing control method of CMG. The system always holds only one equilibrium point. Meanwhile, the frequency synchronization and power sharing control of system are obtained autonomously without any communications. Based on this fundamental research, some decentralized schemes would be constructed for the four-quadrant operating of cascaded system with an unique equilibrium point.

### Table II

<table>
<thead>
<tr>
<th>Parameter</th>
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<tr>
<td>$V^*$ (V)</td>
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<td>$Z_{\text{line}}$ (Ω)</td>
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<tr>
<td>$f'/f$ (Hz)</td>
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<td>$Z_{\text{load}}$ (Ω)</td>
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<td>$m$ (rad/sec.w)</td>
<td>0.01</td>
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<td>9.4+j4.0 in [5, 10]s</td>
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### References


