A Decentralized Control for Cascaded Inverters in Grid-connected Applications

Lang Li, Student Member, IEEE, Yao Sun, Member, IEEE, Hua Han, Member, IEEE, Guangze Shi, Student Member, IEEE, Mei Su, Member, IEEE, Minghui Zheng

Abstract—In this letter, a decentralized control for cascaded inverters is introduced, in which one inverter is controlled as a current source and the others are controlled as voltage sources. The power sharing and synchronization of the inverters are realized without high-bandwidth communication. Meanwhile, it is robust against the voltage sag/well and frequency deviation of grid. Finally, the feasibility of the proposed method is verified by simulation and experimental results.

Index Terms—Cascaded inverters, decentralized control, power sharing control.

I. INTRODUCTION

The cascaded H-bridge inverters have been widely studied and applied in middle- and high-voltage level power network [1]. They are early used for high-voltage motor drivers, STATCOM, and flexible AC transmission systems (FACTS) [2]. Recently, they have been extended to distributed generation, such as AC-stacked PV generation, energy storage system and micro-grids [3–4].

In the past, the centralized control methods [5-6] were widely used for grid-connected cascaded inverters. However, these methods depend on real-time communication networks and powerful centralized controller, which will lead to the reduced reliability due to communication failure, and higher capital costs. Moreover, it becomes more difficult for the centralized methods to be implemented when the number of modules is large.

Recently, there has been some interests on the distributed control strategies [7-8]. A novel power regulation controller for the grid-connected cascaded inverters is designed in [7], but the communication is always needed to maintain the synchronization of system. In addition, the decentralized control methods have also been presented for cascaded H-bridge inverters [9-12]. The control schemes could be classified into two categories according to operation modes. In the islanded mode, [9-10] proposed the inverse power factor droop control for power sharing and autonomous synchronization of system. In grid-connected mode, [11-12] have been presented without any communication, in which all modules are controlled as voltage sources. However, the methods are sensitive to the variations of grid voltage and may lead to instability. Thus, a more robust control approach against the grid voltage variations should be developed.

To address the above concerns, a hybrid current-voltage control scheme is proposed for grid-connected cascaded inverters in this letter. Compared to centralized methods, the proposed scheme have advantages of no need of high bandwidth communications. Therefore, it is more attractive in the applications where the number of inverter modules is large, and the distance between the modules is long. The main features of the proposed scheme are summarized as follows: 1) the phase synchronization of all modules is realized by using the common current information, thus the decentralized manner is obtained; 2) It is robust against the grid voltage sag, distortion and frequency deviations.

II. ANALYSIS OF THE PROPOSED CONTROL SCHEME

A. Equivalent models of grid-connected cascaded inverters

Figure 1 shows the configuration of the grid-connected cascaded inverters, which consists of n H-bridge modules. \(V_i\) and \(\delta_i\) represent the voltage amplitudes and phase angle of the \(i^{th}\) module. \(I_{\text{line}}\) and \(\delta_{\text{line}}\) are grid-connected current amplitudes...
and phase-angle. \(V_g\) and \(\delta_g\) are the voltage amplitudes and phase angle of the utility grid. \(|Z_{\text{line}}|\) and \(\theta_{\text{line}}\) are the grid impedance amplitudes and phase angle. Only the inverter (inverter \#1) near to the grid is controlled as a current source \(I_{\text{line}} \leq \delta_{\text{line}}\). All other inverters \((i = 2, 3, \ldots, n)\) are controlled as voltage sources \(V_i \leq \delta_i\).

### B. Proposed control strategy

Assume that the active and reactive power requirement of the grid are \(P_g\) and \(Q_g\), which are expressed as

\[
P_g = \frac{1}{2} V_{i_{\text{line}}} \cos(\delta_i - \delta_{\text{line}})
\]

\[
Q_g = \frac{1}{2} V_{i_{\text{line}}} \sin(\delta_i - \delta_{\text{line}})
\]

The proposed decentralized control is a hybrid current-voltage control scheme, which is depicted in Fig. 1.

**For the inverter \#1**, its current reference is \(I_{\text{line}} \leq \delta_{\text{line}}\). From (1) and (2), \(I_{\text{line}}\) and \(\delta_{\text{line}}\) are

\[
I_{\text{line}} = \frac{2P_g}{V_g \cos \phi}
\]

\[
\delta_{\text{line}} = \delta_i - \phi
\]

where \(\phi = \arctan(Q_g/P_g)\). Then, the line current is regulated to track \(I_{\text{line}} \leq \delta_{\text{line}}\) by a proportional-resonance (PR) controller [13].

**The other inverters \#i \((i = 2, 3, \ldots, n)\)** are controlled as voltage sources, and the corresponding reference voltage is \(V_i \leq \delta_i\). Assume that the output active power and reactive power reference of \(i^{\text{th}}\) module are \(P^*\) and \(Q^*\), respectively. Then we have

\[
\delta_i = \delta_{\text{line}, i} + \phi
\]

\[
V_i = \frac{2P^*}{I_{\text{line}, i} \cos \phi}
\]

where \(\delta_{\text{line}, i}\) and \(I_{\text{line}, i}\) are the real-time phase angle and amplitudes of line current. Then, the output voltage of the \(i^{\text{th}}\) module is controlled by the double-loop voltage-current controller [13-14]. Since they could be obtained locally by each inverter, these inverters are controlled in decentralized manners.

In the proposed control frame, the inverter \#1 is responsible for regulating the grid current. Meanwhile, the others are controlled as voltage sources to maintain the system synchronization and power regulations according to the common current.

### C. Steady-state analysis

Due to the series structure, the output currents of all inverters are completely the same, (7) is obtained in the steady-state.

\[
\delta_{\text{line}, 2} = \delta_{\text{line}, 1} = \cdots = \delta_{\text{line}, n} = \delta_{\text{line}}
\]

\[
I_{\text{line}, 2} = I_{\text{line}, 1} = \cdots = I_{\text{line}, n} = I_{\text{line}}
\]

From (5) and (6), we have

\[
\delta_i = \delta_{\text{line}} = \cdots = \delta_n = \delta_g
\]

\[
V_i = V_1 = \cdots = V_n
\]

That is to say, these inverters could keep pace with the grid. In this study we let \(P^* = P_g/n\) and \(Q^* = Q_g/n\). Combining (9)-(10), \(P^*_i = V_{i_{\text{line}}} \cos \phi\) and \(Q^*_i = V_{i_{\text{line}}} \sin \phi\), the following equations are derived

\[
P_i = P_g - \sum_{i=2}^{n} P_i = P^*
\]

\[
Q_i = Q_g + Q_{\text{line}} - \sum_{i=2}^{n} Q_i > Q^*
\]

where \(Q_{\text{line}}\) is the reactive power loss of the grid impedance.

In practice, grid voltage may change within contain limits. Then \(P_g\) and \(Q_g\) will change accordingly. Thus, from (13)-(14), the power rating of inverter \#1 will be determined by the fluctuation ranges of grid and safety margin.

### D. Stability analysis

From Fig. 1, the model of system is expressed as

\[
L \frac{dI_{1,i}}{dt} = u_{out,i} - u_g + \sum_{i=2}^{n} u_i
\]

\[
L_f \frac{dI_{f,i}}{dt} = u_{out,i} - u_i
\]

\[
C \frac{du_i}{dt} = i_{L,i} - i
\]

where \(L\) is the grid inductor, \(L_f\) and \(C\) are the inductor and capacitance of LC filter, the subscript \(i = 2, 3, \ldots, n\). \(u_g\) is the grid voltage, \(i\) is the grid current, \(u_{out}\) is the output voltage of the inverter \#i, \(i_{L,i}\) is the filter inductor current of inverter \#i.

The PR controller for *inverter \#1* is expressed as

\[
u_{out,1} = k_p (i' - i) + x_{1,1}
\]

\[
\dot{x}_{1,1} = x_{1,2}
\]

\[
\dot{x}_{1,2} = -2 \omega_c k_p i - \omega^2 x_{1,1} - 2 \omega x_{1,2}
\]

where \(k_p\) and \(k_s\) are the proportional and resonance coefficient of inverter \#1. \(\omega_c\) is the cut-off frequency, \(\omega\) is the synchronous angle frequency of grid, \(i'\) is the reference grid current.

The voltage-current dual closed loop controllers for the *inverter \#i \((i = 2, 3, \ldots, n)\)* are expressed as

\[
u_{out,i} = k_p k_s (u'_i - u_i) + k_p x_{n,i-1} - k_p i_{L,i}
\]

\[
\dot{x}_{i,1} = x_{i,2}
\]

\[
\dot{x}_{i,2} = -2 \omega k_p u_i - \omega^2 x_{i,1} - 2 \omega x_{i,2}
\]
where $k_p$, $k_p$, and $k_v$ are the control coefficients of PR controller. $u^*$ and $i_{r,c}^*$ are the reference capacitor voltage and inductor current.

Because this system is a periodic system, the sinusoidal analysis method is applied [15] to prove the stability of the system. Let $u^* = \text{Re}(V_e e^{j\omega t})$, $i = \text{Re}(I e^{j\omega t})$, $u_{out,i} = \text{Re}(V_{out,i} e^{j\omega t})$, $i_{L,i} = \text{Re}(I_{L,i} e^{j\omega t})$, $x_{1,i} = \text{Re}(X_{1,i} e^{j\omega t})$, $x_{2,i} = \text{Re}(X_{2,i} e^{j\omega t})$, $x_{3,i} = \text{Re}(X_{3,i} e^{j\omega t})$, $x_{4,i} = \text{Re}(X_{4,i} e^{j\omega t})$, where $\text{Re}()$ returns the real part of the complex argument.

Combining (15)-(17) yields (18).

\[
egin{align*}
    j\omega LI + \frac{d}{dt}V_{out,i} &= -V_e + \sum_{n=2}^\infty V_i \\
    j\omega X_{1,i} + \dot{X}_{1,i} &= X_{2,i} \\
    j\omega X_{2,i} + \dot{X}_{2,i} &= -2\omega k_p(j\omega l + I) - \omega^2 X_{1,i} - 2\omega X_{2,i} \\
    j\omega L_i I_{L,i} + \dot{I}_i &= V_{out,i} - V_i \\
    j\omega C_I V_I + \frac{dV_I}{dt} &= I_{L,i} - I_i \\
    j\omega X_{n,i} + \dot{X}_{n,i} &= X_{n+1,i} \\
    j\omega X_{n+1,i} + \dot{X}_{n+1,i} &= -2\omega k_p(j\omega V_I + V_I) - \omega^2 X_{n,i} - 2\omega X_{n+1,i}
\end{align*}
\]

Further, the small signal model of system is expressed as

\[
\dot{I} = \left(\frac{k_p - j\omega}{L}\right)I + \frac{1}{L}\dot{X}_{1,i} + \frac{1}{L} \sum_{n=2}^\infty V_i \\
\dot{X}_{1,i} = \dot{X}_{2,i} - joX_{1,i} \\
\dot{X}_{2,i} = \frac{2\omega k_p}{L} \left(2\omega k_p + j\omega\right)I - \left(2\omega + j\omega\right)X_{1,i} - \frac{2\omega k_p}{L} \sum_{n=2}^\infty V_i \\
\dot{I}_{L,i} = \left(\frac{k_p}{L} + \frac{1}{C}\right)\dot{I}_i - \left(k_p + j\omega\right)I_{L,i} + \frac{k_p}{L} \dot{X}_{n,i} \\
\dot{V}_I = \frac{1}{C}\left(I + \frac{1}{L}\dot{I}_i - jow\right) \\
\dot{X}_{n,i} = \dot{X}_{n+1,i} - jowX_{n,i} \\
\dot{X}_{n+1,i} = \frac{2\omega k_p}{C}I - \frac{2\omega k_p}{C}X_{n+1,i} - (2\omega + j\omega)X_{n,i}.
\]

For brevity, (19) is rewritten as matrix form

\[
X = AX
\]

where the state variable vector $X$ and the system matrix $A$ are

\[
\begin{align*}
\text{Real} & \quad \text{Imaginary} \\
-3000 & \quad 0 \\
-2000 & \quad 0 \\
-1000 & \quad 0 \\
0 & \quad 0 \\
1000 & \quad 0 \\
2000 & \quad 0 \\
3000 & \quad 0 \\
\end{align*}
\]

![Fig. 2. Root locus diagram as parameter changes.](image)

The root locus of the closed loop system is depicted based on the experimental tests, the effect of $k_p$, $k$, $k_u$, and $L$ are studied. As seen in Fig. 2, all eigenvalues are always in the left half-plane. That is to say, the system is stable when the appropriate control and physics parameters are selected.

### III. SIMULATION RESULTS

![Fig. 3. Simulation model consisting of six modules.](image)

| TABLE I |

<table>
<thead>
<tr>
<th>Method</th>
<th>Parameters</th>
<th>Values</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed scheme</td>
<td>$V_e$ (V)</td>
<td>600</td>
<td>$P^*$ (W)</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>$l_{line}$ (H)</td>
<td>1.2e-3</td>
<td>$Q^*$ (Var)</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>$V_e$ (V)</td>
<td>600</td>
<td>$P^*$ (W)</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>$l_{line}$ (H)</td>
<td>1.2e-3</td>
<td>$\omega^*$ (rad/s)</td>
<td>100\pi</td>
</tr>
</tbody>
</table>
The circuit level simulation model consisting of six modules are carried out on the platform of MATLAB/SIMULINK using SimPowerSystems (see Fig.3). This simulation is implemented based on the method in [11] and the proposed scheme. The associated parameters are listed in Table I.

![Fig. 4. Simulation results.](image)

**A. Case1: the method in [11]**

This simulation with 10% grid voltage sag at t=5s is performed based on the method in [11]. The active power and reactive power sharing results are depicted in Fig. 4(a1) and (b1). The reactive power is negative to maintain the system stable operation [11] before t=5s. The frequencies are shown in Fig. 4(c1). Therefore, it is concluded that the system is unstable under the grid voltage sag condition based on the method in [11].

**B. Case2: the proposed scheme**

Under the same setup as case 1, this simulation is implemented with the proposed scheme under the 10% grid voltage sag at t=5s. The waveforms of active power, reactive power and frequency are shown in Fig. 4(a2), (b2) and (c2), respectively. The reactive power is always positive, which indicates that the proposed scheme holds the capability of reactive power compensation to the grid. When the grid voltage sag occurs after t=5s, the system could always maintain the system stable and inject the desired powers into the grid. The simulation results show that the proposed scheme is more robust compared to the method in [11] under the same grid voltage sag condition.

**C. Case3: fault-tolerant operation with the proposed scheme**

This simulation with module #6 suddenly lost at t=5s is carried out based on the proposed scheme. When a failure occurs in module #6, the bypass method [16] is applied. The waveforms of active and reactive power are shown in Fig. 5(a) and (b), in which the power shortage of module #6 is supplied automatically by module #1 because it is a free variable. Therefore, the proposed scheme can realize the fault-tolerant operation within a certain extent.

![Fig. 5. Simulation results when module #6 is by-passed.](image)

**IV. EXPERIMENTAL RESULTS**

The prototype setup of the grid-connected cascaded inverters shown in Fig. 6 is built in the lab, which is consisting of three modules. The control diagram of the proposed scheme is shown in Fig. 1. The experimental parameters are listed in Table II.

![Fig. 6. Prototype setup of the grid-connected cascaded inverters.](image)

**TABLE II**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_x^0 ) (V)</td>
<td>60</td>
<td>( P^* ) (W)</td>
<td>20</td>
</tr>
<tr>
<td>( I_{line} ) (H)</td>
<td>5e-3</td>
<td>( Q^* ) (Var)</td>
<td>4</td>
</tr>
<tr>
<td>( L_f ) (H)</td>
<td>0.6e-3</td>
<td>( C_f ) (( \mu )F)</td>
<td>20</td>
</tr>
<tr>
<td>( V_{d1} ) (V)</td>
<td>40</td>
<td>( V_{d2},V_{d3} ) (V)</td>
<td>30</td>
</tr>
</tbody>
</table>

**A. Case1: normal grid condition**

![Fig. 7. Experimental waveforms under the normal-grid condition.](image)
This experiment is performed under the normal grid condition. The experimental waveforms of voltage and current are presented in Fig. 7, in which the output voltages of module #2 and #3 are always in-phase with grid. The active power sharing results are shown in Fig. 8(a), in which the active powers of module #1 are higher than others due to the active power losses in experiment. Figure 8(b) shows that the reactive powers of module #1 are more than others to compensate the absorbed reactive power of line. From the experimental results, the proposed scheme can inject the desired powers into grid and maintain the self-synchronized operation.

**B. Case2: grid voltage sag condition**

This experiment with the 10% grid voltage sag is carried out. The measured waveforms are depicted in Fig. 9, in which the output voltage of module #2 and #3 reduces while the line current increases after the grid voltage sag occurring. The test results are shown in Fig. 10(a) and (b), in which the powers of module #1 are increased as the line current increases. The experimental results indicate that the proposed scheme is adaptive to the grid voltage sag condition.

**C. Case3: grid frequency deviations**

This experiment with the grid frequency deviation (-1Hz) is implemented. The voltage and current waveforms are shown in Fig. 11. The frequencies are shown in Fig. 12(a), in which all modules always maintain synchronization with the grid even under such large disturbances. The active and reactive power allocations are shown in Fig. 12(b) and (c). Clearly, the proposed scheme is robust against the grid frequency deviation.

**D. Case4: DC input voltage sag condition**
This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TIE.2019.2945266, IEEE Transactions on Industrial Electronics

This test with grid inductance variation (from 5mH to 6.2mH) is performed. The related waveforms are shown in Fig. 15. The active and reactive powers are depicted in Fig. 16(a) and (b), respectively. Compared to the results in case 1, the reactive power of module #1 is increased to compensate more reactive powers when the grid inductor increases. As seen, the proposed scheme is suitable for the grid impedance variations.

**F. Case 6: the method in [11]**

This test is carried out with the control method in [11], and the corresponding experiment waveforms are shown in Fig. 17. When the 10% grid voltage sag occurs, the grid current begins to divergent until the overcurrent protection is triggered. Compared to case 2, it is concluded that the proposed scheme is insensitive to the grid voltage condition. That is to say that the proposed scheme is more robust than the method [11].

**G. Case 7: grid harmonics condition**

The test under the distorted grid voltage (THD=5.26%) is implemented. The experimental waveforms are shown in Fig. 18. As seen, the proposed scheme is still feasible under the condition of the grid distortion.

**V. CONCLUSION**

This letter proposes a decentralized control scheme of the grid-connected cascaded inverters, where the synchronization operation is realized via the common line current information. Due to the decentralized control, the number of cascaded modules is unlimited in theory. Experimental results show that the cascaded system could operate normally under the abnormal-grid conditions (grid voltage sag, distortion and frequency deviation). That is to say the method is robust against the grid voltage variations to a certain extent. Based on these features, it has the potential to be extended to the medium/high...
voltage level photovoltaic, storage and STATCOM cascaded systems, where the number of series modules is very large.

**APPENDIX**

\begin{equation}
X = [X_1, X_2, \ldots, X_n]
\end{equation}

where \(X_i = [I_{x_{i,1}}, \hat{x}_{i,2}, \ldots, \hat{x}_{i,n}]\).

\begin{equation}
A = \begin{bmatrix}
A_1 & A_2 & \cdots & A_2 \\
A_3 & A_4 & 0 & 0 \\
A_3 & 0 & 0 & A_4
\end{bmatrix}
\end{equation}

where

\[
A_1 = \begin{bmatrix}
-k_p - j\omega & 1/L & 0 \\
L & 0 & -j\omega & 0 \\
2\omega k_p k_r & (2\omega k_r + \omega)^2 & -2\omega j - j\omega & 0
\end{bmatrix}
\]

\[
A_2 = \begin{bmatrix}
0 & 1/L & 0 & 0 \\
0 & 0 & -2\omega k_r/L & 0 \\
0 & 0 & 0 & -2\omega k_r/C_f
\end{bmatrix}
\]

\[
A_3 = \begin{bmatrix}
0 & 0 & 0 \\
0 & 1/C_f & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
A_4 = \begin{bmatrix}
-k_p/L & (k_p + \omega/L)k_r/L & 0 \\
-(k_p + \omega/L)k_r/L & 1/C_f & 0 \\
-k_p/L & 0 & 0 \\
2\omega k_r/C_f & -j\omega & 0 \\
0 & 0 & 0 \\
-j\omega & -2\omega j - j\omega
\end{bmatrix}
\]

**REFERENCES**


