A Decentralized SOC Balancing Method for Cascaded-type Energy Storage System

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Abstract—As unbalance state of charge (SOC) of storage units usually leads to the decrease of lifetime, SOC balancing control is essential. In this paper, a decentralized SOC balancing method is proposed to balance the SOC of cascaded-type energy storage systems. Since the method does not rely on any communication, it possesses higher reliability. As is well known, SOC is a slowly changing variable compared to other variables such as voltage and current. Thus, the studied system has obvious two-time scale characteristic. In addition, the stability analysis of the system based on the singular perturbation theory is carried out. Simulation and experiments are performed to verify the validity of the proposed SOC balancing scheme.

Index Terms—Decentralized control, SOC balancing, Cascaded converter, Storage units.

I. INTRODUCTION

MICROGRID, as a future smart decentralized power network, has been developed with high penetration of renewable generations [1]-[2]. It offers an effective solution to integrate distributed renewable sources, loads and energy storage systems [3]-[4]. Meanwhile, the intermittence poses a challenge to renewable generation application in microgrid, which worsens the power quality and decreases the stability of the utility grid.

An energy storage system (ESS) is adopted to restrain the power fluctuation of these intermittent resources [5]-[6]. Also, ESSs bring some benefits such as energy shifting and peak shaving of power system [7]-[10]. Actually, the different initial SOC values and mismatched output power would cause the unbalancing SOC problem, which may lead to the over charge and discharge actions. These factors would decrease the lifetime of ESSs.

To deal with the unbalanced SOC problem, the SOC levels of ESSs should be coordinated over long time scales. According to the interfacing converter topologies, the ESS can be divided into two categories: the paralleled-type and the cascaded-type ones. The paralleled-type ESS has been widely studied recently due to its flexibility as well as plug and play characters, and thus it is widely used in low voltage microgrid. The control strategies for SOC balancing in paralleled-type ESS have been extensively investigated, which are generally divided into three categories: (a) centralized [11]-[12] (b) distributed and [13]-[15] (c) decentralized control [16]-[19]. In [11]-[12], the centralized coordinated secondary control strategies are presented for the SOC balancing among ESSs. However, the centralized control would be invalid when the centralized communication fail. To reduce communication burden and improve network reliability, the distributed SOC balancing control approaches are studied in [13]-[15]. In [13], a droop control with distributed communication is studied to balance SOC. The modification on droop curves can coordinate change the output power of each storage unit for achieving SOC convergence. In [14-15], a dynamic consensus algorithm for balancing discharge rates is proposed based on secondary control. The algorithm proposed in [15] provides virtual impedance loops, which achieves SOC balancing by adjusting virtual impedance. However, these distributed control methods still rely on the communications which would cause additional cost. Moreover, decentralized approaches based on droop control [16-19] have been proposed to deal with the unbalancing SOC problem without any communication. In [16-18], the output power can be regulated through adjusting droop coefficients to balance SOC of storage units autonomously. In [19], with modifying the droop offsets, the units with lower SOC levels provide smaller share of load demand, and the units with larger SOC values may supply more power for load demand. Besides no communication network, these decentralized control methods also have several advantages including simplicity, flexibility, and reliability. Although these aforementioned methods are useful, it is not enough for the applications of cascaded-type ESS.

Cascaded energy storage system (CESS) are mainly applied to large-capacity energy storage application. The cascaded converter topology are emerging in middle and high level voltage storage system application due to its merits in using low-voltage semiconductor switches, producing low output-voltage harmonic distortion and inherent modularity. It can make the CESS acquire simple voltage-scaling properties without expensive and bulky transformers. In addition, the interfacing converters can provide an ability to achieve SOC balancing among all energy storage units (ESUs) without using additional balancing circuits [20]. In this occasion, the SOC balancing control is also indispensable. Some control methods [21-23] are used to adjust active power of each storage unit in CESS. The strategy to control the SOC of ESS in [21] enables power compensation with smaller energy capacity and with less effect to the compensation result. In [22], a control method for
SOC balancing of the multiple storage unit is researched with cascading PWM converters. To manage the SOC values of ESS, phase-phase SOC-balancing control and inter-phase SOC-balancing control are analyzed in [23]. The aforementioned SOC control methods are dependent on a centralized controller which needs high-bandwidth communication. When the number of ESUs is very large, the cost of the high-bandwidth communication would become a heavy load. Moreover, the possible communication failures may affect the system’s normal operation. All of the aforementioned drawbacks would limit the development of the centralized control.

The decentralized control is more attractive because it is not affected by the communication dependence and limitation of ESU number, and has higher reliability. Thus, it has been applied in cascaded converter systems. For example, an inverter power factor droop control for power sharing is proposed [24]. However, its application is limited to a certain load. To overcome the limitation an $f-P/Q$ droop control is proposed [25]. In addition, some other decentralized control methods have been presented for cascaded PV [26] and cascaded STATCOM [27]. Considering the SOC for CESS, the SOC balancing control methods are almost based on centralized control. The drawbacks mentioned above lead the inapplicability in large scale CESS. Therefore, this paper focuses on studying a decentralized method to achieve SOC balance.

Up to our knowledge, no other literature has been published for balancing SOC for CESS in a decentralized manner except for [28]. In this paper which is the extended version of [28], a decentralized SOC balancing control algorithm is proposed for CESS. Under the proposed control strategy, each unit in CESS is controlled independently only by using its local information, and the output power of each unit can be regulated according to its SOC level. Compared with [28], a comprehensive analysis is introduced to analyze the steady state performance and system stability based on the singular perturbation theory, due to the two time scale characteristic of the researched system. Moreover, the SOC balancing is theoretically proved in steady state and the detailed parameters selecting ranges are given. Finally, more simulation results and experiment tests are presented to validate the feasibility of the proposed control.

Compared with the existing SOC balancing methods, as shown in Table I, the benefits of the proposed control strategy are concluded as follows:
1. The SOC balancing among all ESUs is achieved in CESS autonomously.
2. Without communication dependence, the proposed decentralized control has the advantages of higher reliability, stronger fault tolerance and lower construction cost.
3. The proposed control has good extensibility, and its application is not limited by the number of ESUs.
4. Theoretical proof about SOC balancing is provided based on the singular perturbation theory.

Under the proposed control strategy, each unit in CESS is controlled independently only by using its local information, and the output power of each unit can be regulated according to its SOC level. The remaining of this paper is organized as follows. Section II presents the design of SOC balancing control method. Section III illustrates the stability analysis of the proposed SOC balancing control based on the singular perturbation theory and small signal analysis. Section IV and Section V present simulation and experimental results, respectively. Section VI concludes the paper.

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II. PROPOSED SOC BALANCING CONTROL

A. Equivalent Model of Islanded CESS

Fig. 1 illustrates an islanded CESS consisting of $n$ ESUs, interfacing converters and local loads. In this configuration, the interfacing converters are single phase DC/AC converters, all of which are connected in series to energize the loads. The by-pass switch is used to protect the ESU if the SOC of one ESU is not in its safe range or the ESU has failed.

For simplicity, each DC/AC converter is regarded as a controlled voltage source (CVS). Thus, the output voltage of the $i$-th converter is denoted as $V_{i \theta}$, and the coupling point voltage is represented by $V_{p e^{j \theta}}$. The relationship between the coupling point voltage and the voltages of all controlled voltage sources is expressed as follows

$$V_{pe^{j \theta}} = \sum_{i=1}^{n} V_{i e^{j \theta}}$$  \hspace{1cm} (1)$$

From Fig. 1, the output active power $P_i$ and the reactive power $Q_i$ of the $i$-th controlled voltage source are derived as follows
\[ P + jQ = V_i e^{j\theta_i} \left( V'_e e^{j\theta'_e} / |Z_{\text{load}}| e^{j\theta_{\text{load}}} \right) \]  

(2)

where the \(|Z_{\text{load}}|\) and \(\theta_{\text{load}}\) are the load impedance magnitude and angle.

According to (1)-(2), the power transmission characteristics of the \(i\)-th controlled voltage source are obtained as

\[ P_i = \frac{V_i}{|Z_{\text{load}}|} \sum_{j=1}^{N} V_j \cos(\theta_j + \theta_{\text{load}} - \theta_i) \]  

\[ Q_i = \frac{V_i}{|Z_{\text{load}}|} \sum_{j=1}^{N} V_j \sin(\theta_j + \theta_{\text{load}} - \theta_i) \]  

(3) (4)

According to the definition of SOC, the SOC of the \(i\)-th ESU is expressed as follows

\[ \text{SOC}_i = \text{SOC}_{0i} - \frac{1}{C_i} \int i_{in}^{\text{in}} dt \]  

(5) (6)

where \(\text{SOC}_{0i}\) is the initial SOC, \(C_i\) and \(i_{in}^{\text{in}}\) denote the nominal capacity and terminal current of the \(i\)-th ESU, respectively.

\[ \text{SOC}_{c_{ij}} = \min_{1 \leq j \leq N_i} \left( \text{SOC}_{0ij} C_j \right) \]  

\[ C_i = \min \left( \text{SOC}_{0ij} C_j \right) + \min \left( 1 - \text{SOC}_{0ij} \right) C_j \]  

(7) (8)

where \(\text{SOC}_{0ij}\) and \(C_j\) represent the initial SOC value and capacity of the \(j\)-th battery cell in an \(i\)-th storage unit, respectively. \(N_i\) represents the number of the battery cell in storage units.

Differentiating (5) with respect to time and combining with (6), the states of the battery model can be derived

\[ \dot{V}_i = \frac{1}{C_i} \left( I_{in}' - \frac{V_i}{Z_i} \right) \]  

\[ \dot{S}\text{SOC}_i = -\frac{i_{in}^{\text{in}}}{C_i} \]  

(9) (10)

As seen in section III, the proposed control only rely on the SOC information of each storage unit. Therefore, the chemical dynamics of ESU model is ignored in the following analysis.

Neglecting the power losses in the inverter, the output power of inverter \(P_i\) yields,

\[ P_i \approx P_{in}^{\text{in}} = V_i^{\text{in}} \cdot I_{in}^{\text{in}} \]  

(11)

where \(P_{in}^{\text{in}}\) is the output power of the \(i\)-th ESU.

Combining (6) and (11), the relationship between SOC and output power of \(i\)-th ESU is obtained, which plays a significant role in the following analysis.

\[ \text{SOC}_i = \text{SOC}_{0i} - \int i_{in}^{\text{in}} dt = \text{SOC}_{0i} - \int \frac{P_{in}^{\text{in}}}{E_{\text{max},i}^{\text{in}}} dt \]  

(12)

where \(E_{\text{max},i}^{\text{in}} = V_i^{\text{in}} \cdot C_i\).

To realize SOC balancing, it is vital to know accurate SOC information. However, as the objective of the paper is not the SOC estimation, the Coulomb Counting approach to estimate SOC is employed for simplicity. Certainly, to get more accurate SOC, advanced SOC estimation methods can be used. For more details, the interested readers may refer to [34].

C. Proposed SOC Balancing Control Method

![Four-quadrant working cases in CESS](image)

According to reference [25], the mechanisms of autonomous synchronization in islanded cascaded-type system are different due to the load impedance characteristic. Thus, in this paper, four quadrant operation for storages, as illustrated in Fig. 3, should be taken into account. It includes: discharging with resistive-inductive load (I), charging with resistive-inductive load (II), charging with resistive-capacitive load (III), and discharging with resistive-capacitive load (IV).

First of all, the rated power of the power sources should be
pre-designed to avoid overload with considering the load demand. This is the precondition for ensuring the safe operation of the system. Then in order to achieve SOC balancing, the proposed control scheme is designed as follows,

\[
\begin{align*}
\omega_i & = \omega_0 + sgn(Q_j)(m_i P_i - n_i SOC_i) \\
V_i & = \varphi(E_{max,i})V^*
\end{align*}
\]

(13)

where the sgn(·) denotes sign function. \(\omega_0\) and \(V_i\) are the angular frequency reference and voltage amplitude reference of the \(i\)-th controlled voltage source, respectively. \(\omega_0\) represents the value of \(\omega\) with no load. \(V^*\) is the nominal voltage amplitude value of the bus voltage. \(m_i\) is the adjustment coefficient of the proposed controller which is designed to be inversely proportional to their control gains, i.e. \(m_i E_{max,i} = K\) where \(K\) is a constant. \(n_i\) is a SOC weighted coefficient to modify the droop control adaptively. For achieving SOC balancing, we choose \(n_1 = \ldots = n_n = N\). The \(\varphi(E_{max,i})\) is designed as follow

\[
\varphi(E_{max,i}) = E_{max,i}/\sum_{j=1}^{n} E_{max,j}
\]

(14)

When the CESS has reached the steady state, the frequency synchronization and SOC balancing can be achieved. Then we can obtain

\[
m_i sgn(Q_j) P_i = m_j sgn(Q_j) P_j
\]

(15)

Define that \(\varphi_i(\varphi_j)\) is the phase angle difference between the output voltage \(V_i(V_j)\) and the common current \(I\) of the \(i\)-th \((j\)-th\) CVS. Combine (13), (14) and (15), then

\[
KV^* I sgn(Q_j) \cos \varphi_i = KV^* I sgn(Q_j) \cos \varphi_j
\]

(16)

From (16), it is obvious that \(sgn(\sin \varphi_j) \cos \varphi_j = sgn(\sin \varphi_i) \cos \varphi_j\). Then \(\varphi_i = \varphi_j\) or \(\varphi_i = -\varphi_j\) can be obtained. As the \(\varphi_i = -\varphi_j\) is the unfeasible solution, the power angle can keep the same for all CVS in steady state. Thus the bus voltage amplitude equals the sum of voltage amplitude of all the CVS i.e. the nominal voltage amplitude reference of the bus voltage.

\[
V_p = \sum_{i=1}^{n} V_i = V^*
\]

(17)

D. Double Control Loop Design

To obtain a better tracking performance, the double control loop is applied in this paper, which comprises of voltage control and current control. As the voltage reference calculated by proposed method is an AC variable, the proportional-resonant (PR) is more applicable than PI control to track the AC variables [33]. The voltage and current control are respectively designed as follows:

\[
G_i(s) = k_{PV} + \frac{2k_{PV} \cdot \omega_{PV} \cdot s}{s^2 + 2\omega_{PV} \cdot s + \omega_{PV}^2}
\]

(18)

\[
G_i(s) = k_{PV} + \frac{2k_{IV} \cdot \omega_{IV} \cdot s}{s^2 + 2\omega_{IV} \cdot s + \omega_{IV}^2}
\]

(19)

where \(k_{PV}\) and \(k_{PV}\) are the proportional and resonant parameters of the voltage control, respectively. The \(k_{PV}\) and \(k_{IV}\) are the proportional and resonant parameters of the current control, respectively. In addition, the \(\omega_r\) is the resonant frequency and the \(\omega_{PV} (\omega_{IV})\) represents the cut-off frequency of the voltage control (current control).

III. STABILITY OF PROPOSED SOC BALANCING CONTROL

A. Singular perturbation theory

As the studied system is a two-time scale system, it can be expressed into the following singular perturbation system [36, 37].

\[
\begin{align*}
\frac{dx}{d\tau} & = f(x, y) \\
\frac{dy}{d\tau} & = g(x, y)
\end{align*}
\]

(20)

where \(x, y\) and \(e\) are the fast variable, the slow variable and the time scale parameter, respectively.

Rewrite the model in the stretched time scale \(t = \tau / e\), as follows

\[
\begin{align*}
\frac{dx}{dt} & = f(x, y) \\
\frac{dy}{dt} & = e g(x, y)
\end{align*}
\]

(21)

where \(\tau\) and \(t\) are referred to as the slow and the fast time scales, respectively. The models in (20) and (21) represent the corresponding slow and fast system respectively.

In the slow system, it is usually considered that the fast variable achieves quasi-steady state while the slow variable is still varying [37]. Setting \(e = 0\) in (20), the outer system is expressed as

\[
\begin{align*}
0 & = f(x, y) \\
\frac{dy}{dt} & = 0
\end{align*}
\]

(22)

where \(x_s\) is the quasi-steady-state value of \(x\).

While in the fast system, the slow variables act several ten times slower than the fast variables. The variable \(y\) can be considered as a constant parameter. Define \(\tilde{x} = x - x_s\), then the boundary layer system is expressed as

\[
\begin{align*}
\frac{d\tilde{x}}{dt} & = f(\tilde{x}, y) \\
\frac{dy}{dt} & = 0
\end{align*}
\]

(23)

B. System model

From (12)-(13), the dynamic equations (24) of the \(i\)-th controlled voltage source can be written

\[
\begin{align*}
\frac{d\theta_i}{dt} & = \theta_0 + sgn(Q_i)(m_i P_i - N \cdot SOC_i) \\
\frac{dSOC_i}{dt} & = -m_i P_i / K
\end{align*}
\]

(24)

Define \(\hat{\theta}_i = \theta_i - \theta_0\), \(e = \sqrt{e}\), then

\[
\begin{align*}
\frac{d\hat{\theta}_i}{dt} & = \theta_0 + sgn(Q_i)(m_i P_i - N \cdot SOC_i) - \omega_h \\
\frac{dSOC_i}{dt} & = -e m_i P_i
\end{align*}
\]

(25)
where \( \theta_i = \omega_{i0} + \theta_{i0} \), \( \omega_{i0} \) is a quasi-steady state value of \( \omega_i \), and \( \theta_{i0} \) depends on the selection of reference phase angle and \( \varepsilon \) is very small.

Rewrite the model (25) in the time scale: \( \tau = \varepsilon t \), then

\[
\begin{align*}
\frac{d\hat{\theta}_i}{d\tau} &= \omega_i + \text{sgn}(Q_i)(m_i \cdot P_i - N \cdot SOC_i) - \theta_i, \\
\frac{dSOC_i}{d\tau} &= -m_i P_i
\end{align*}
\] (26)

\[\text{C. Analysis on the outer system}\]

Setting \( \varepsilon = 0 \) in (26), the outer system of the \( i \)-th controlled voltage source is expressed as

\[
\begin{align*}
\omega_i &= \omega_{i0} + \text{sgn}(Q_i)(m_i \cdot P_i - N \cdot SOC_i), \\
\frac{dSOC_i}{dt} &= -m_i P_i
\end{align*}
\] (27)

where \( P_i \) and \( Q_i \) are the quasi-steady state value of \( P_i \) and \( Q_i \), respectively.

In the quasi-steady state, by combining the outer systems of both \( i \)-th and \( k \)-th controlled voltage sources we have

\[
\frac{dSOC_i}{dt} + N \cdot SOC_i = \frac{dSOC_k}{dt} + N \cdot SOC_k
\] (28)

Define \( \gamma SOC_i - SOC_k \), and (28) can be rewritten as

\[
\gamma \frac{dz}{dt} + z = 0
\] (29)

where \( \gamma = 1/N \).

The solution of (29) is derived as

\[
z(t) = z(0)e^{-\gamma t}
\] (30)

Clearly, \( z \) converges to zero as time goes to infinity. The convergence rate depends on \( N \). The larger the \( N \), the faster the convergence rate is. Thus, in the steady state we have

\[
SOC_i = SOC_k
\] (31)

From the analysis above, the SOC balancing of all controlled voltage sources can be achieved in steady state under the proposed control.

\[\text{D. Analysis on the boundary layer system}\]

For simplicity, we assume that \( \varphi(E_{\text{max}, i}) = \varphi(E_{\text{max}, j}) = 1/n \),

where \( n \) is the number of CVS in CESS. Setting \( \varepsilon = 0 \) in (25), we can regard the \( \omega_i \) and \( SOC_i \) as invariants. Substitute (3) into (25), then

\[
\frac{d\tilde{\theta}_i}{d\tau} = \omega'_i + \text{sgn}(Q_i)m_i V_{2}^2 n^2 |Z_{\text{load}}| \sum_{j=1}^{n} \cos(\tilde{\theta}_j - \tilde{\theta}_i + \theta_{\text{load}})
\] (32)

where \( \omega'_i = \omega_{i0} - \omega_{i0} - N \cdot SOC_i \).

To prove the stability of the boundary layer system (32), the small signal analysis is used [36].

It is reasonable to assume the state variable in equilibrium point are \( [\tilde{\theta}_{i1}, \tilde{\theta}_{i2}, \ldots, \tilde{\theta}_{in}] \), where the \( \tilde{\theta}_{in} \) is the quasi-steady state value of \( \tilde{\theta}_i \). By linearizing (32), the small-signal equations are derived as:

\[
\frac{d\Delta \tilde{\theta}_i}{d\tau} = -\text{sgn}(Q_i)\frac{m_i V_{2}^2}{n^2 |Z_{\text{load}}|} \sum_{j=1}^{n} \sin(\tilde{\theta}_j - \tilde{\theta}_i + \theta_{\text{load}})(\Delta \tilde{\theta}_j - \Delta \tilde{\theta}_i)
\] (33)

where \( Q_i = \frac{V_{2}^2}{n^2 |Z_{\text{load}}|} \sum_{j=1}^{n} \sin(\tilde{\theta}_j - \tilde{\theta}_i + \theta_{\text{load}}) \).

Rewrite (33) in the matrix form

\[
\Delta \hat{\theta} = A \cdot \Delta \hat{\theta}
\] (34)

where \( A \) is the coefficient matrix.

\[
A = -B \cdot L
\] (35)

where \( B = \text{diag}[b_1, b_2, \ldots, b_n] \), \( b_i = \text{sgn}(Q_i)\frac{m_i V_{2}^2}{n^2 |Z_{\text{load}}|} \).

According to gershgorin circle theorem, the eigenvalues of the system coefficient matrix \( A \) fall within the gershgorin regions.

\[
\lambda_i(A) \in G = \bigcup_{j=1}^{n} G_j
\] (36)

\[
G_i(A) = \left\{ z \in \mathbb{C} \left| \| z + \sum_{j=1}^{n} b_j l_j \| \leq R_i \right. \right\}, \quad i = 1, 2, \ldots, n
\] (37)

where \( R_i \) is the \( i \)-th gershgorin circle radius, \( R_i = \sum_{j=1}^{n} |b_j l_j| \).

To ensure stability, the eigenvalues of the coefficient matrix \( A \) should be on the left half plane, i.e., all gershgorin circles should be on the left half plane. From (33)-(37), if all \( b_j l_j > 0 \), the system will be stable.

As \( m_i V_{2}^2 |Z_{\text{load}}| \) is positive, the stability condition is simplified as

\[
\text{sgn}(Q_i)\sin(\theta_{\text{load}} + \tilde{\theta}_i - \tilde{\theta}_j) > 0
\] (38)

From (38), the system is stable, if the following condition holds.

\[
\max_{i, j, k, l \in \{1, 2, \ldots, n\}} |\tilde{\theta}_i - \tilde{\theta}_j - \tilde{\theta}_k - \tilde{\theta}_l| < |\theta_{\text{load}}|
\] (39)

From (3), the quasi-steady state active powers of \( i \)-th and \( k \)-th can be expressed as

\[
P_{ia} = \frac{V_{2}^2}{n^2 |Z_{\text{load}}|} \sum_{j=1}^{n} \cos(\tilde{\theta}_j - \tilde{\theta}_i + \theta_{\text{load}})
\] (40)

\[
P_{ka} = \frac{V_{2}^2}{n^2 |Z_{\text{load}}|} \sum_{j=1}^{n} \cos(\tilde{\theta}_j - \tilde{\theta}_k + \theta_{\text{load}})
\] (41)

Combining (40) and (41), we have

\[
\sin(\frac{\tilde{\theta}_{ia}}{2}) = \frac{-P_{ia} - P_{ka} n^2 |Z_{\text{load}}|}{2V_{2}^2 \sum_{j=1}^{n} \cos(\tilde{\theta}_j + \theta_{\text{load}} + 2\theta_{\text{load}})}
\] (42)

where \( \tilde{\theta}_{ia} = \tilde{\theta}_i - \tilde{\theta}_a \).

In the quasi-steady state, all controlled voltage sources share
the same frequency. From (13), we can obtain
\[ P_i - P_j = N (SOC_i - SOC_j) / m_i \]  \hfill (43)

Substituting (43) in (42) results in
\[ \sin \left( \frac{\theta_{i_k} - \theta_{j_k}}{2} \right) = \frac{-N (SOC_i - SOC_j) n^2 |Z_{load}|}{2m |V|^2 \sum_{j=1}^{n} \sin \left( \frac{\theta_{i_j} + \theta_{j_j} + 2 \theta_{i_k}}{2} \right)} \]  \hfill (44)

From (44), if \( SOC_i = SOC_k \), then \( \theta_{i_k} = 0 \), i.e., all phase angles are the same. If \( SOC_i \neq SOC_k \), it is suggested to choose appropriate \( m_i \) and \( N \) to satisfy (39). So the feasible range of \( m \) and \( n \) should be discussed. Taking the stability condition and allowable frequency deviation into condition, the boundary condition can be derived as follows
\[ \sin \left( \max_{i,j \in [1,2,\ldots,n]} \left| \theta_{i_k} - \theta_{j_k} \right| / 2 \right) < \sin \left| \theta_{load} / 2 \right| \]  \hfill (45)

\[ |\Delta \omega| = |m_i P_i - N \cdot SOC_i| \leq \Delta \omega_{\text{max}} \]  \hfill (46)

where the \( \Delta \omega_{\text{max}} \) is the maximum allowable frequency deviation which is often set to be 1 in the microgrid system [39]. The power \( P_i \) has been limited in \([-P_{\text{max}}^*, P_{\text{max}}^*]\), and the \( P_{\text{max}}^* \) is the nominal rated power, which is set to be 1500W. The safe range of \( SOC \) is set to be \([0.1, 0.9]\).

![Fig. 4 Feasible range of m and N](image)

To facilitate visualization of the feasible range of \( m \) and \( N \), the results of Fig. 4 are based on small signal model around one quasi-steady state point \((t=10s \& Z_{load}=4+j8)\) and the assumption that all the CVS have the same capacities.

From Fig. 4, the \( N \) and \( m \) are specified in the \( N \)-axis and \( m \)-axis and the boundaries corresponding to stability and frequency deviation. It is easy to see that a large \( m \) and small \( N \) can extend the stable range of the system. But it is worth mentioning that a very large \( m \) may lead to unallowable frequency deviation and a very small \( N \) may decrease the SOC convergence rate. So selecting a point further away from the boundary line and a lager \( N \) for faster SOC convergence rate can make the system achieve better performance. Therefore, the selection of \( m_i \) and \( N \) is a trade-off among the stability, frequency deviation and the SOC convergence rate.

As in whole, according to the proposed control function (13), the intercept of \( P \cdot f \) curve are modified by the local SOC information, which is to adjust the output power of ESUs by the different output voltage angles. The basic principle is described in Fig. 5. In the discharging mode, higher SOC energy storage units result in greater power and lower SOC ones result in smaller power. Likewise, in the charging mode, units with lower SOC are injected with more power, and ones with larger SOC are injected with less power. Based on this principle, the SOC difference will be decreased and the SOC balancing will be achieved ultimately.

![Fig. 5 Principles of the proposed SOC balancing control method: (a) resistive-inductive load, (b) resistive-capacitive load](image)

**Remark:** The distinctions of the proposed method in this paper from the existing decentralized methods [16-19] are demonstrated in the following: (1) Previous works focus on the parallel-type ESS, while this paper researched on the cascaded-type ESS. (2) The control algorithms in [16-19] are based on droop control. In contrast, our algorithm is derived from inverse droop control. (3) The SOC balancing is proved based on multi-time scale analysis. Theoretical proof about SOC balancing has not been reported in the literatures.

<table>
<thead>
<tr>
<th>Case</th>
<th>Control Coefficient</th>
<th>Storage unit Capacities(A·h)</th>
<th>DC Voltage(V)</th>
<th>Voltage Reference(V)</th>
<th>Frequency Reference(Hz)</th>
<th>Load Characters(Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( m_i ) ((10^4))</td>
<td>( m_j ) ((10^4))</td>
<td>( N ) ((10^4))</td>
<td>( C_{C1} )</td>
<td>( C_{C2} )</td>
</tr>
<tr>
<td>Case A</td>
<td></td>
<td>2   2   2   2   5   4   4   4 160  300</td>
<td>50</td>
<td>4+j8 and 4-j8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case B</td>
<td></td>
<td>2   2   2   2   5   4   4   4 160  300</td>
<td>50</td>
<td>4+j8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case C</td>
<td></td>
<td>2   2   2   2   5   4   4   4 160  300</td>
<td>50</td>
<td>4+j8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case D</td>
<td></td>
<td>4/3 2   4   5   6   4   4   2 160  300</td>
<td>50</td>
<td>4+j8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case E</td>
<td></td>
<td>2   2   2   2   0.5 2   4   4 160  300</td>
<td>50</td>
<td>4+j8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
IV. SIMULATION RESULTS

To validate the proposed SOC balancing control method, the simulation model based on three controlled voltage sources are developed in the MATLAB/Simulink environment. Fig. 6 shows the detailed control diagram of proposed method, which contains power calculation loop, proposed control block, and the control block of voltage and current control loops. The proposed control is to provide a voltage reference based on the local measured active and reactive power information. Then the voltage and current controllers use PR control to track the voltage reference. The system parameters are listed in Table II and the parameters of double control loop are listed in Table III.

A. Case A: Simulation results of SOC balancing in four-quadrant operations

In this case, the control gains $m_1 = m_2 = m_3 = 0.0002$, and $N=0.5$ are selected for the three energy storage units. Different initial SOC values of each energy storage units are selected as follows. $SOC_{01}, SOC_{02}$ and $SOC_{03}$ are set to be $90\%, 80\%, 70\%$ for the discharging mode and $10\%, 20\%, 30\%$ for the charging mode, respectively. The resistive-inductive load $Z_L = 4+j8 \Omega$, and resistive-capacitive load $Z_L = 4+j8 \Omega$ are used in this case.

Fig. 7 (a), (b), (c) and (d) show the SOC and the output power of the energy storage units operating in the first, second, third, and fourth quadrant, respectively. As shown in Fig. 7, the deviation $\Delta SOC = SOC_{max} - SOC_{min}$, and $\Delta P = P_{max} - P_{min}$. From the results in Fig. 7, the proposed SOC balancing controller perform well and the SOC balancing as well as power sharing are achieved eventually.

B. Case B: Simulation results in mode switching between discharging and charging

The mode switching between the charging and discharging processes is tested in this section. The parameters of the simulated system are the same with those in Case A. The mode switching is enabled at 1000s.

![Diagram](image-url)
Fig. 8 Performance of transferring from discharging to charging process with proposed method: (a) SOC; (b) Active Power.

Fig. 8 presents the waveforms of energy storage units operating from the discharging mode to the charging mode under the resistive-inductive load. In the discharging mode, the SOC values of the three units are convergent, and the deviation $\Delta$SOC is reduced from 20% to 1.9%. After switching to the charging mode, the convergence of SOC is still guaranteed, and the deviation $\Delta$SOC gradually reduces to zero in the steady state. Moreover, the deviation $\Delta P$ has little fluctuations during the mode switching process.

Fig. 9 shows the test results from charging mode to discharging mode. As shown in Fig. 9, the variation process of SOC and power are similar to the mode switching from the discharging to the charging. The SOC balancing and the power sharing are gradually achieved throughout the whole process and the fluctuations of mode switching are acceptable. From the simulation results above, the proposed SOC balancing control has good performance during mode switching.

C. Case C: Simulation tests under discharging mode with load characteristics changing.

The dynamic response during the load changing from inductive-resistive to capacitive-resistive one is shown in Fig. 10. In this case, all of the three energy storage units operate in the discharging mode. At $t=1000$s, the load is changed from the inductive-resistive to the capacitive-resistive one. It shows that the convergences of SOC and power sharing are not affected by the changing of the load characteristics.

Fig. 10 Performance of load characteristic changing from inductive-resistive to capacitive-resistive in discharging mode: (a) SOC; (b) Active Power.

D. Case D: Simulation tests of proposed method with different capacities of ESU
In order to show the effectiveness of proposed method with different capacities of ESU, the capacities of ESU#1, ESU#2, and ESU#3 are set to 6, 4, and 2A·h, respectively. Meanwhile, the droop coefficient are changed to be 4/3×10^{-4}, 2×10^{-4}, and 4×10^{-4}, respectively.

Fig. 11 shows the test results of SOC and output active power when the capacities of ESUs in CESS are different. In steady state, the SOC balancing has been already achieved and the output power values of ESUs are measured as 1.11, 0.738, and 0.364 kW, respectively. It is clearly shown that the power allocation can be described as an equation: \( P_1 : P_2 : P_3 \approx 3 : 2 : 1 \)

As that the DC voltage are set to the same value, the output power is in proportion to its capacities, which can be derived as follows: \( P_1 : P_2 : P_3 \approx C_1 : C_2 : C_3 \). From the simulation result, the SOC balancing and proportional power sharing are proved to be achieved with different capacities of ESU.

**E. Case E: Simulation results of comparison of ESS with and without SOC balancing control.**

The section provides the simulation results of comparison of ESS with and without SOC balancing control. The initial SOC values SOC_01, SOC_02 and SOC_03 are set to be 50%, 45%, 40%. Generally, the safe SOC range of Li-FePO_4 battery is between 10% and 90%. From Fig. 12, the ESS without SOC balancing control works in unsafe range at 868s, while the one with SOC balancing control still works in safe range. To some extent, the depth of charge-discharge can be decreased with the proposed control, i.e. the life of ESS can be increased [40].

![Fig. 12 Simulation results of comparison of ESS with and without SOC balancing control. (a) Without SOC balancing control, (b) with the SOC balancing control](image)

**V. EXPERIMENT RESULTS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Item Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>V/(I_{ref})</td>
<td>Voltage reference</td>
<td>25 V/50 Hz</td>
</tr>
<tr>
<td>(C_{e1} = C_{e2})</td>
<td>Unit capacities</td>
<td>2Ah</td>
</tr>
<tr>
<td>(i_{ph})</td>
<td>Output voltages of unit</td>
<td>32V</td>
</tr>
<tr>
<td>(n_{r1n2})</td>
<td>Control coefficients</td>
<td>3.26-4</td>
</tr>
<tr>
<td>(N)</td>
<td>SOC weighted coefficient</td>
<td>0.15</td>
</tr>
<tr>
<td>(Z_{load})</td>
<td>Inductive-resistive (RL) load</td>
<td>9.39+4.05 (\Omega)</td>
</tr>
<tr>
<td>(Z_{load})</td>
<td>Capacitive-resistive (RC) load</td>
<td>12.53-j5.99 (\Omega)</td>
</tr>
</tbody>
</table>

In this section, a cascaded-type islanded system with two storage units is employed to validate the proposed control method, which is shown in Fig. 13 and the main circuits of the experiment system is shown in Fig. 14. Each storage unit has 10 battery cells connected in series. The nominal voltage, capacity, impedance, max charge current and max discharge current of battery cell are 3.2V, 2Ah, ≤1.3mΩ, 10C and 30C, respectively. The inverters are controlled by DSP program (TMS320F28335), to implement the proposed SOC balancing control algorithm. The variables of active power and SOC are noted with 10s sampling period. The parameters of experiments are listed in Table IV. It is noted that the different SOC initial values under two different type loads are set to verify the validation of the SOC balancing control under various cases.

![Fig. 13 Experimental setup of two cascaded storage units](image)

![Fig. 14 Main circuits of the experiment system](image)
The proposed SOC balancing control is realized. Until time $t=9s$, both the SOC balancing and the power sharing are achieved. It is noted that the different SOC initial values under two different type loads are set to verify the validation of the SOC balancing control under various cases, which has no effect on whether we can obtain our conclusion or not.

In charging mode, the DC voltage source is under the constant current control, which charges ESS and energizes loads. Figs. 19 and 20 show the measured waveforms under RL loads in charging mode. The proposed SOC balancing control is enabled at $t=11s$, and the initial SOC values are: SOC$_{01}=0.23$ and SOC$_{02}=0.27$. As shown from Fig. 20, under the proposed control, the injected power and the SOC are balancing in the steady state, which validate the feasibility of the propose control in charging mode.
In this paper, we propose a decentralized SOC balancing scheme for cascaded-type energy storage system. This scheme achieves SOC balancing of energy storage units through adjusting P-f curves. Without communication dependence, the proposed decentralized control obtains higher reliability and lower construction cost. Meanwhile, the proposed control has good extensibility, and its application is not limited by the number of ESUs. Moreover, based on the singular perturbation theory, the SOC steady state values are proved to be equal. The stability of the system is proved and a sufficient condition for the stable operation is provided. Both the simulation and experiment results have validated the proposed method.

VI. CONCLUSIONS

In this paper, we propose a decentralized SOC balancing scheme for cascaded-type energy storage system. This scheme achieves SOC balancing of energy storage units through adjusting P-f curves. Without communication dependence, the proposed decentralized control obtains higher reliability and lower construction cost. Meanwhile, the proposed control has good extensibility, and its application is not limited by the number of ESUs. Moreover, based on the singular perturbation theory, the SOC steady state values are proved to be equal. The stability of the system is proved and a sufficient condition for the stable operation is provided. Both the simulation and experiment results have validated the proposed method.

VII. REFERENCES


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